*mh m-1kl∖P .... , \*—r» ſ r fnk∖P*

— 4- —— 1 —, which is equal to *P -*—τ~ *{ b3 — —* ) T, t∕irτ^r∕2, 1 *m3 ∖ r J* 2,

*, m—* I ∕ ,, *τn D- ∖ e2*

and finally to — ? —— ( Z-3- -— ) ~.

*' m ∖ r J 2*

Therefore the focal diſtancc of refracted rays is *x* = φor ∖ *r ∕ 2*

This conſiſts of two parts. The firſt *φ* is the focal distance of an infinitely ſlender pencil of central rays, and the w—, ∕ ∖ gl . ,

other — e∙2 ■ ^3- Z\*3— ~ J ~ is the aberration ariſing

from the ſpherical figure of the refracting ſurface.

Our formula has thus at laſt put on a very ſimple form, and is vaſtly preferable to Dr Smith’s for practice.

This aberration is evidently proportional to the ſquare of the ſemi-aperture, and to the ſquare of the diſtance *φ* : but, in order to obtain this ſimplicity, ſeveral quantities were neglected. The aſſumption of the equality of AX to — is the firſt ſource of error. A much more accurate value of it

*2ae2 e2 ■*

would have been ——, for it is really ≡≡ AX. If

4 *a* ⅛ e 3 2 *a—*A X

*ei*

*for AX* we ſubſtitute its approximated value ~,we ſhould have AX = r, ≈ —τ 5. To have uſed this value would not have much complicated the calculus ; but it did not occur to us till we had finiſhed the inveſtigation, and it would have required the whole to be changed. The operation in page 346. col. 2. par. 2. is another ſource of error. But theſe errors are very inconſiderable when the aperture is moderate. They increaſe for the moſt part with an increaſe of aperture, but not in the proportion of any regular function of it ; ſo that we cannot improve the for­mula by any manageable proceſs, and muſt be contented with it. The errors are preciſely the ſame with thoſe of Dr Smith’s theorem, and indeed with thoſe of any that we have ſeen, which are not vaſtly more complicated.

As this is to be frequently combined with ſubſequent operations, ” we ſhorten the expreſſion by putting θ for *τn—1 ∕* m ⅛J2 ∖ e2

*• ( k3-~∙* J—» Then φ2θ will expreſs the aberra­tion of the firſt refraction from the focal diſtance of an infi­nitely ſlender pencil; and now the focal diſtance of refracted rays, is *f = φ —* φ2θ.

If the incident rays are parallel, *r* becomes infinite, and

θ = *——P —.* But in this caſe *k* becomes = —, and — 7723 -2 *a φ*

*m — I ma ni1 ai m—*I

= , and ® = ““—, and ?“ e becomes-; v2 × 5-

*m a m—*1 —1) *m*

I *C €*

× × -\*^- , ≈ —, t—■. This is the aberration of extreme parallel rays.

We muſt now add the refraction of another ſurface.

*Lemma 2.* If the focal diſtance AG be changed by a ſmall quantity Gg, the focal diſtance AH will alſo be changed by a ſmall quantity Hh, and we ſhall have

*m* × ΛG2 : AH2 = Gg : Hh

Draw Mg, M*h,* and the perpendiculars Gi, Hk. Then, becauſe the sines of the angles of incidence are in a constant ratio to the sines of the angles of refraction, and the incre­ments of theſe ſmall angles are proportional to the increments of the sines, theſe increments of the angles are in the ſame conſtant ratio. Therefore,

We have the angle CMg to HMh as m to I.

Now *Gg : G i* — AG : A M,

and *Gi : k k = m* × *AG : H*A,

and *bk* : Hh = MA : AH :

therefore Gg ; Hh = *m* × AG2 : AH2.

The eaſieſt and moſt perſpicuous method for obtaining the aberration of rays twice refracted, will be to consider the firſt refraction as not having any aberration, and deter­mine the aberration of the ſecond refraction. Then con­ceive the focus of the firſt refraction as ſhifted by the aber­ration, This will produce a change in the focal diſtance of the ſecond refraction, which may be determined by this Lemma.

Prop. II. Let AM, BN(fig. 7.) be two ſpherical surfaces, including a refracting ſubſtance, and having their centres C and *c* in the line AG. Let the ray *aA* paſs through the centres, which it will do without refraction. Let another ray. mM, tending to G, be refracted by the firſt ſurface into MH, cutting the ſecond ſurface in N, where it is farther refracted into NI. It is required to de­termine the focal diſtance BI ?

It is plain that the sine of incidence on the ſecond ſur­face is to the sine of refraction into the ſurrounding air as I *to m.* Alſo BI may be determined in relation to BH, by means of BH, Nx, Br, and — , in the ſame way that AH was determined in relation to AG, by means of AG, MX, AC, and *m.*

Let the radius of the ſecond ſurface be *b,* and let *e* ſtill expreſs the ſemi-aperture, (becauſe it hardly differs from Nx). Alſo let α be the thickness of the lens. Then obſerve, that the focal diſtance of the rays refracted by the firſt ſurface, (neglecting the thickneſs of the lens and the aberration of the firſt ſurface), is the diſtance of the radiant point for the ſecond refraction, or is the focal diſtance of rays incident on the ſecond ſurface. In place of *r* therefore we muſt take *φ ;* and as we made *k =* —, in order to abbreviate the calculus, let us now make l *≡ 1/b —* and make j^=∙^-*mh*

, 1 I 4 τ n, . , r a m — X

as we made . Laſtly, in place of θ = —7-

*a m j r mi*

*∕ m L∖e2 ſ\* v ∕ *l2 ∖e2*

Gj- v) 2»make 6=G√~1 ) w V ~ Γ = ~

*m ∖* i> *J* 2

Thus we have got an expreſſion ſimilar to the other; and the focal diſtance BI, after two refractions, becomes BI = *f-ΓL*

But this is on the ſuppoſition that BH is equal to φ, whereas it is really φ *—* φ2θ — α. This muſt occaſion a change in the value juſt now obtained of BI. The ſource of the change is twofold. 1st, Becauſe, in the value \_L—2, we muſt put —, and becauſe we muſt -do the ſame in the fraction -. In the ſecond place, when the value of BH is diminiſhed by the quantity φ2 + α, Bl will ſuffer a change in the proportion determined by the 2d Lem­ma. The firſt difference may ſafely be neglected, becauſe the value of θ is very ſmall, by reaſon of the coefficient e2/2 being very ſmall, and alſo becauſe the variation bears a very ſmall ratio to the quantity itſelf, when the true value of φ