canals fc and *co.* There are ſeveral other conditions equal­ly neceſſary to which this lax reaſoning will not apply, ſuch as the direction of the whole remaining gravitation in any point F. This muſt be perpendicular to the ſurface, &c. &c. Nor will this mode of inveſtigation aſcertain the eccentricity of the ſpheroid without a moſt intricate proceſs. We muſt therefore take the ſubject more generally, 2nd ſhow the proportion and directions of gravity in every point of the ſpheroid. We need not, however, again demonſtrate that the gravitation of a particle placed any where without a perfect ſpherical ſhell, or a ſphere conſiſting of concentric ſpherical ſhells, either of uniform density, or of denſities varying according to ſome function of the radius, is the ſame as if the whole matter of the ſhell or sphere were collected in the centre. This has been demonſtrated in the article Astronomy. We need only remind the reader of ſome conſequences of this theorem which are of continual uſe in the preſent inveſtigation.

1st, The gravitation to a ſphere is proportional to its quantity of matter directly, and to the ſquare of the diſtance of its centre from the gravitating particle inversely.

2d, If the ſpheres be homogeneous and of the same denſity, the gravitations of particles placed on their ſurfaces, or at diſtances which are proportional to their diameters, are as the radii ; for the quantities of matter are as the cubes of the radii, and the attractions are inverſely as the ſquares of the radii ; and therefore the whole gravitations are as r3/r2, or as r.

3d, A particle placed within a ſphere has no tendency to the matter of the ſhell which lies without it, becauſe its ten­dency to any part is balanced by an oppoſite tendency to the oppoſite part. Therefore,

*4th,* A particle placed any where *within* a homogeneous ſphere gravitates to its centre with a force proportional to its diſtance from it.

It is a much more difficult problem to determine the gra­vitation of particles to a ſpheroid. To do this in general terms, and for every ſituation of the particle, would require a train of propoſitions which our limits will by no means admit ; we muſt content ourſelves with as much as is neceſ­ſary for merely aſcertaining the ratio of the axes. This will be obtained by knowing the ratio of the gravitation at the pole to that at the equator. Therefore

Let N*m*SqN (fig. 2) be a ſection through the axis of an oblate homogeneous ſpheroid, which differs very little from a ſphere. NS is the axis, *mq* is the equatorial dia­meter, O. is the centre, and NMSQ is the ſection of the inſcribed ſphere. Let P be a particle ſituated at any distance without the ſphere in its axis produced ; it is re­quired to determine the gravitation of this particle to the whole matter of the ſpheroid ?

Draw two lines PAC, PBD, very near to each other, cutting off two ſmall arches AB, CD ; draw GAa, HBb, IC*c,* KD*d,* perpendicular to the axis ; alſo draw OE and AL perpendicular to PAC, and OF perpendicular to PD, cutting PC in f*.* Join OA.

Let OA, the radius of the inſcribed ſphere, be r, and OP the diſtance of the gravitating particle be *d,* and Mm, the elevation of the equator of the ſpheroid, or the elliptici­ty, be *e.* Alſo make AE = *x,* and OE = y, — r2 — x2.

Then AE — BF = *x* and Ff = y, =

√r1 x2

Suppoſe the whole figure to turn round the axis OP. The little ſpace AB *ba* will generate a ring of the redun­dant matter ; ſo will CD *dc.* This ring may be conſidered as conſiſting of a number of thin rings generated by the revolution of A*a.* The ring generated by A*a* is equal to a parallelogram whoſe baſe is the circumference deſcribed by A and whoſe height is A*a.* Therefore let *c* be the circumference of a circle whoſe radius is 1. The ring will be A*a* × *c* × AG. But becauſe *ma*N is an arch of an ellipſe, we have M*m* : A*a* = MO : AG = *r* : AG, and A*a* = M*m×* AG/r, = e/r × AG. Therefore the ſurface of this ring is = *ce/r(AG2)*.

We have ſuppoſed the ſpheroid to be very nearly ſpherical, that is, *e* exceedingly ſmall in compariſon of r. This being the caſe, all the particles in Aa, and conſequently all the particles in the ring generated by the revolution of A*a,* will attract the remote particle P with the ſame force that A does very nearly. We may ſay the ſame thing of the whole matter of the ring generated by the revolution of ABba. This attraction is exerted in the direction PA by each individual particle. But every action of a particle A is accompanied by the action of a particle A in the di­rection PA'. These two compoſe an attraction in the di­rection PO. The whole attraction in the directions ſimilar to PA is = *c* × eAG2/rPA2 × GH, for GH meaſures the number of parallel plates of which the ſolid ring is compoſed.

This being decompoſed in the direction PG is = *c × e/r* × (AG2 × PG)/PA2. But AG2/PA2, and PG/PA = PE/PO. Therefore the attraction of the ring, eſtimated in the direction PO, is = c × e/r × (OE2 × PE)/PO3 × GH.

Farther, by the nature of the circle, we have HG : AB = AG : AO ; alſo AB : BL = AO : OE. But PA : (AC × PO)/PA. Therefore AB : BL = AO : (AG × PO)/PA, = AO. PA : PO. AG

Alſo BL : LA = EO : EA,

And LA : Ff = PA : *Pf, =* ultimately PA : PE. There­fore, by equality, HG : Ff = AG . AO . PA . EO . PA : AO . PO. AG EA . PE.

Or HG : Ff = EO. PA2 : PO. EA . PE.

And HG = Ff × (EO . PA2)/(PO . PE . EA).

Now ſubſtitute this value of HG in the formula expressing the attraction of the ring. This changes it to *c e/r × (O*E2. PE)/PO3 × (OE . PA2)/(PO . PE . EA) × Ff, or c(e/r) × (OE3 . PA2)/PO4 . EA) × Ff. In like manner, the attraction of the ring generated by the revolution of CD*dc is c(e/r) ×* (OE3 . PC2)/(PO4 . EA) × Ff. Therefore the attraction of both is c(e/r)*×* Ff × OE3/(PO4 . EA)× PA2 + PC2, = c(e/r) × Ff[y3/(d4 . x)] × PA = + PC2. But PA2 + PC2 = 2 PE2 + 2EA2, = 2PE2 + 2x2. Therefore the attraction is 2*c(e/rd4)* × Ff(y3/x) × PE2 + x2. But F*f = y, = x/y* × x. Therefore Ff(y3/x) = (x/y)x × y3/x, = y2x,