their mouths. This may appear absurd, and is certainly very paradoxical ; but it is a fact eſtabliſhed on the moſt unexceptionable authority. One inſtance fell under our own obſervation. The low water mark at ſpring tide in the harbour of Alloa was found by accurate levelling to be three feet higher than the top of the stone pier at Leith, which is ſeveral feet above the high water mark of this har­bour. A little attention to the motion of running waters will explain this completely. Whatever checks the motion of water in a canal muſt raise its ſurface. Water in a canal runs only in conſequence of the declivity of this ſurface : (See River). Therefore a flood tide coming to the mouth of a river checks the current of its waters, and they accumulate at the mouth. This checks the current farther up, and therefore the waters accumulate there alſo ; and this checking of the ſtream, and conſequent rising of the waters, is gradually communicated up the river to a great diſtance. The water riſes everywhere, though its ſurface ſtill has a ſlope. In the mean time, the flood tide at the mouth passes by, and an ebb ſucceeds. This muſt accelerate even the ordinary courſe of the river. It will more remarkably acce­lerate the river now raised above its ordinary level, becauſe the declivity at the mouth will be ſo much greater. There­fore the waters near the mouth, by accelerating, will sink in their channel, and increaſe the declivity of the canal be­yond them. This will accelerate the waters beyond them ; and thus a ſtream more rapid than ordinary will be produ­ced along the whole river, and the waters will sink below their ordinary level. Thus there will be an ebb below the ordinary ſurface as well as a flood above it, however ſloping that ſurface may be.

Hence it follows, that we cannot tell what is the natural ſurface of the ocean by any obſervations made in a river, even though near its mouth. Yet even in rivers we have regular tides, ſubjected to all the varieties deduced from this theory.

We have ſeen that the tide is always proportional to MS. It is greateſt therefore when the moon is in conjunction or oppoſition, being then S*s,* the ſum of the ſeparate tides produced by the sun and moon. It gradually decreaſes as the moon approaches to quadrature ; and when ſhe is at Q or *q,* it is SO, or the difference of the ſeparate tides. Sup­posing Ss divided into 1000 equal parts, the length of MS is expreſſed in theſe parts in the fourth column of the fore­going table, and their differences are expreſſed in the fifth column.

We may here obſerve, that the variations of the tides in equal ſmall times are proportional to the sine of twice the diſtance of the place of high water from the moon. For ſince M *n* is a constant quantity, on the ſuppoſition of the moon’s uniform motion, M*v* is proportional to the variation of MS. Now Mn : Mv = MC :CI = I : sin. 2*y,* and M*n* and MC are constant quantities.

Thus we have ſeen with what eaſe the geometrical conſtruction of this problem not only explains all the intereſting circumſtances of the tides, but alſo points them out, almoſt without employing the judgment, and exhibits to the eye the gradual progreſs of each phenomenon. In theſe reſpects it has great advantages over the very elegant algebraic analyſis of Mr Bernoulli. In that process we advance almoſt without ideas, and obtain our solutions as detached facts, without perceiving their regular ſeries. This is the uſual pre-eminence of geometrical analyſis ; and we regret that Mr Bernoulli, who was eminent in this branch, did not rather employ it. We doubt not but that he would have shown ſtill more clearly the connection and gradual progreſs of every particular. His aim, however, being to inſtruct thoſe who were to calculate tables of the different affections of the tides, he adhered to the algebraic method. Unfor­tunately it did not preſent him with the eaſieſt formulæ for practice. But the geometrical conſtruction which we have given ſuggeſts ſeveral formulæ which are exceedingly simple, and afford a very ready mode of calculation.

The fundamental problems are to determine the angle *sOh* or *mOh,* having *mOs* given ; and to determine MS.

Let the given angle mOs be called *a ;* and, to avoid the ambiguity of algebraic ſigns, let it always be reck­oned from the neareſt ſyzigy, so that we may always have *a* equal to the ſum of *x* and *y.* Alſo make a2 =

S2 × sin.2 2*a*

-- ir-r ; 7 , which represents the Sc2/SM2 M2 + S2 + 2M × S × cos. 2*a* SM2

of fig. 4. or sin.2 2y, and make *p = — 1--»  j, p = M + S × cos. 2a*

Sc

which is the expreſſion of Sc/Mc of that figure, or of tan. *2y.* Then we ſhall have,

**1.** Sin. y = ∕t v 1 λ'∖ For we ſhall have coſ. 2y =

V 2

∕ Γ τ3 r 2 1 1 Γ . I —√I-√\*

v i—But sin. *y =*  cos. *2y — -s*

2 2 2

and sin. y = *∕ LDL-L·-- —.*

*2.* Tan. y = 7 —-. For becauſe *p* is = tan. y = i 4√1 4/

*2y, V* i 4- *p2* is the ſecant of 2y, and I + *V + p2* : I = p : tan. y.

Theſe proceſſes for obtaining y directly are abundantly ſimple. But it will be much more expeditious and eaſy to content ourſelves with obtaining *2y* by means of the value of its tangent, viz — Or; we may find *x* by

M + S coſ. 2*a*

means of the ſimilar value of its tangent Md/Sd, of fig. 4.

There is ſtill an eaſier method oſ finding both 2*x* and 2y, as follows.

Make M + S : M — S = tan. *a* : tan. *b.* Then *b* is the difference of *x* and y, as *a* is their ſum. For this analogy evidently gives the tangent of half the difference of the angles CSM and GMS of fig. 4. or of 2 *x* and *2y.* There­fore to *a,* which is half the ſum of 2*x* + 2y, add *b,* and we have 2x *= a + b,* or *x =* (a + b)/2, and *y = (a — b)/2.*

By either of theſe methods a table may be readily com­puted of the value of *x or y* for every value of *a.*

But we muſt recollect that the values of S and M are by no means constant, but vary in the inverſe triplicate ratio of the earth’s diſtance from the sun and moon ; and the ratio of 2 to 5 obtains only when theſe luminaries are at their mean diſtances from the earth. The forces correſponding to the perigean medium and apogean diſtances are as follow.

Hence we see that the ratio of S to M may vary from 1,901 : 5,925 to 2,105 : 4,528, that is, nearly from 1 : 3 to 1 : 2, or from 2 : 6 to 2 : 4. The solar force does not vary much, and may be retained as constant without any great error. But the change of the moon's force has great effects on the tides both as to their time and their quantity.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Sun. | | Moon. |
| Apogean |  | 1,901 | 4,258 |
| Medium |  | 2, | 5, |
| Perigean |  | 2,105 | 5,925 |