To find the angle at B, we have only to ſubtract the angle BDA (= 61⁰ 56' ) from 90⁰, and the rem. 28⁰ 4' is the angle ſought. The angle at C is equal to 53⁰ 7'.

3. *By Gunter.*

*1st,* ‘ The extent from 105 to 135, will reach from **35 to** 45 on the line of numbers.’ 2*dly,* ‘ The extent from 85 **to 75,** on the line of numbers, will reach from radius to **61⁰**56', the angle BDA on the line of sines.’ 3dly, ‘ The extent from 50 to 30 on the line of numbers, will reach from radius to angle ADC 36⁰ 53' on the line of sines.’

The foregoing three cases include all the varieties of plane triangles that can happen, both of right and oblique- angled triangles. But beſides theſe, there are ſome other theorems that are uſeful upon many occaſions, or ſuited to ſome particular forms of triangles, which are often more expeditious in uſe than the foregoing general ones ; one of which, for right-angled triangles, as the caſe for which it ſerves ſo often occurs, may be here inſerted, and is as follows.

Case IV. When, in a right-angled triangle, there are given the angles and one leg, to find the other leg, or the hypothenuſe. Then it will,

As radius:

To given leg AB: :

So tang. adjacent the angle A :

To the oppoſite leg BC, and : :

So ſec. of same angle A :

To hypot. AC:

|  |  |
| --- | --- |
| *Example.* In the triangle ABC (fig. 4.), right-angled at B, | |
|
| Given the leg AB = 162 | to find BC and AC. |
| and the angle A = 53⁰ 7' 48⁰ |
| conſeq. the angle C = 36 52 12 |

I. *Geometrically.—*Draw the leg AB = 162 : Erect. the indefinite perpendicular BC : Make the angle A = 53⁰ 1/8, and the side AC will cut BC in C, and form the triangle ABC. Then, by meaſuring, there will be found AC = 270, and BC = 216.

|  |  |  |
| --- | --- | --- |
|  | **2.** *Arithmetically.* |  |
| As radius | = 10 | log. 10'0000000 |
| To AB | = 162 | 2'2o9515o |
| So tang. A | = 53⁰ 7' 48'' | 10'1249372. |
| To BC | = 216 | 2.3344522 |
| So ſec. A | = 53° 7' 48'' | 10'2218477 |
| To AC | = 270 | 2'4313627 |

3. *By Gunter.*

Extend the compasses from 45⁰ at the end of the tan­gents (the radius) to the tangent of 53⁰ ⅛ ; then that extent will reach, on the line of numbers, from 162 to 216, for BC. Again, extend the compaſſes from 36⁰ 52' to 90 on the sines ; then that extent will reach, on the line of num­bers, from 162 to 270 for AC.

*Note,* Another method, by making every side radius, is often added by the authors on trigonometry, which is thus : The given right-angled triangle being ABC, make firſt the hypothenuſe AC radius, that is, with the extent of AC as a radius, and each of the centres A and C, deſcribe arcs CD and AE (fig. 5.) ; then it is evident that each leg will repreſent the sine of its oppoſite angle, viz. the leg BC the line of the arc CD or of the angle A, and the leg AB the sine of the arc AE or of the angle C. Again, making either leg radius, the other leg will repreſent the tangent of its oppoſite angle, and the hypothenuſe the ſecant of the same angle ; thus, with radius AB and centre A deſcribing the

arc BF, BC repreſents the tangent of that arc, or of the angle A, and the hypothenuſe AC the ſecant of the ſame ; or with the radius BC and centre C deſcribing the arc BG, the other leg AB is the tangent of that arc BG or of the angle C, and the hypothenuſe CA the ſecant of the ſame.

And then the general rule for all theſe caſes is this, *viz.* that the ſides bear to each other the ſame proportions as the parts or things which they repreſent. And this is called making every side radius.

SPHERICAL TRIGONOMETRY.

Spherical Trigonometry is the art whereby, from three given parts of a ſpherical triangle, we discover the rest ; and, like plane trigonometry, is either right-angled or oblique angled. But before we give the analogies for the ſolution of the ſeveral caſes in either, it will be proper to premiſe the following theorems :

Theorem I. In all right-angled ſpherical triangles, the ſign of the hypothenuſe : radius : : sine of a leg : sine of its oppoſite angle. And the sine of a leg : radius : : tangent **of** the other leg : tangent of its oppoſite angle.

*Demonstration.* Let EDAFG *(ibid.* fig. 6.) repreſent the eighth part of a ſphere, where the quadrantal planes EDFG, EDBC, are both perpendicular to the quadrantal plane ADFB ; and the quadrantal plane ADGC is per­pendicular to the plane EDFG ; and the ſpherical triangle ABC is right-angled at B, where CA is the hypothenuſe, and BA, BC, are the legs.

To the arches GF, CB, draw the tangents HF, OB, and the sines GM, CI, on the radii DF, DB ; alſo draw BL the sine of the arch AB, and CK the sine of AC ; and then join IK and OL. Now HF, OB, GM, CI, are all perpendicular to the plane ADFB. And HD, GK, OL, lie all in the ſame plane ADGC. Alſo FD, IK, BL, lie all in the ſame plane ADGC. Therefore the right-angled triangles HFD, CIK, ODL, having the equa-angles HDF, CKI, OLB, are ſimilar. And CK : DG : : CI : GM ; that is, as the sine of the hypothenuſe : rad. : : sine of a leg : sine of its oppoſite angle. For GM is the sine of the arc GF, which meaſures the angle CAB. Alſo, LB : DF : : BO : FH ; that is, as the sine of a leg : radius : : tangent of the other leg : tangent of its Opposite angle. Q. E. D.

Hence it follows, that the sines of the angles of any oblique ſpherical triangle ACD (fig. 7.) are to one another, directly, as the sines of the oppoſite ſides. Hence it alſo follows, that, in right-angled ſpherical triangles, having the ſame perpendicular, the sines of the baſes will be to each other, inversely, as the tangents of the angles at the baſes.

Theorem H. In any right-angled ſpherical triangle ABC (fig. 8.) it will be, As radius is to the co-ſine of one leg, ſo is the co-ſine of the other leg to the co-ſine of the hypothenuſe.

Hence, if two right-angled ſpherical triangles ABC, CBD (fig. 7.) have the ſame perpendicular BC, the co-ſines of their hypothenuſes will be to each other, directly, as the co-ſines of their baſes.

Theorem III. In any ſpherical triangle it will be, As radius is to the sine of either angle, ſo is the co-ſine of the adjacent leg to the co-ſine of the oppoſite angle.

Hence, in right-angled ſpherical triangles, having the ſame perpendicular, the co-ſines of the angles at the base will be to each other, directly, as the lines of the vertical angles.

Theorem IV. In any right-angled ſpherical triangle