|  |  |  |  |
| --- | --- | --- | --- |
| The Solution of the Cases of oblique ſpherical Triangles, (fig. 9 and 10.) | | | |
| Caſe | Given | Sought | Solution |
| **I** | Two ſides AC, BC, and an angle A oppo­ſite to one of them. | The angle B oppoſite to the other | As sine BC : line A : : sine AC : sine B (by theorem 1.) Note, this caſe is ambiguous when BC is leſs than AC ; ſince it cannot be determined from the data whether B be acute or obtuſe. |
| 2 | Two ſides AC, BC, and an angle A oppo­ſite to one of them | The included angle ACB | Upon AB produced (if need be) let fall the perpendicular CD ; then (by theorem 4.) rad. : co-ſine AC : : tang. A : co-tang. ACD ; but (by theorem 1.) as tang. BC : tang. AC : : coſine ACD : co-ſine BCD. Whence ACB—ACD ± BCD is known. |
| 3 | Two ſides AC, BC, and an angle oppoſite to one of them | The other side  AB | As rad. : co-ſine A : : tang. AC : tang. AD (by theor. 1 ) and (by theor. 2.) as co-ſine AC : co-ſine BC : : co-ſine AD : co-ſine BD. Note, this and the laſt caſe are both ambi­guous when the first is ſo. |
| 4 | Two ſides AC, AB, and the included angle A | The other side BC | As rad. : co-ſine A : : tang. AC : tang. AB (by theor. 1.) whence AD is alſo known ; then (by theor. 2.) as co-ſine AD : co-ſine BD : : co-ſine AC : co-sine BC. |
| 5 | Two ſides AC, AB, and the included angle A | Either of the other angles, ſuppoſe B | As rad. : co-ſine A : : tang. AC : tang. AD (by theor. 1.) whence BD is known ; then (by theor. 4.) as sine BD : sine AD : : tang. A : tang. B. |
| *6* | Two angles A, ACB, and the side AC betwixt them | The other angle  B | As rad. : co-ſine AB : : tang. A : co-tang. ACD (by theo­rem 4.) whence BCD is alſo known ; then (by theor. 3.) as sine ACD : sine BCD : : co-ſine A : co-sine B. |
| 7 | Two angles A, ACB, and the side AC betwixt them | Either of the other ſides, ſuppoſe BC | As rad. : co-ſine AC : : tang. A : co-tang. ACD (by theo­rem 4.) whence BCD is alſo known : then, as co-ſine BCD : co sine ACD : : tang. AC : tang. BC (by theor. 1.) |
| 8 | Two angles A, B, and a side AC oppoſite to one of them | The side BC oppoſite the other | As sine B : sine AC : : sine A : sine BC (by theorem 1.) |
| 9 | Two angles A, B, and a side AC oppoſite to one of them | The side AB betwixt them | As rad. : co-ſine A : : tang. AC : tang. AD (by theor. 1.) and as : tang. B : tang. A : : sine AD : sine BD (by theorem 4.) whence AB is alſo known. |
| 10 | Two angles A, B, and a side AC oppoſite to one of them | The other angle  ACB | As rad. : co-ſine AC : : tang. A : co-tang. ACD (by theo­rem 4. ) and as co-ſine A : co-ſine B : : sine ACD : sine BCD (by theor. 3.) whence ACB is alſo known. |
| **II** | All the three ſides  AB, AC, and BC | An angle, ſuppoſe A | As tang. 1/2AB : tang.(AC + BC)/s : : tang. (AC — BC)/2 : tang. DE, the diſtance of the perpendicular from the middle of the baſe (by theorem 6.) whence AD is known : then, as tang. AC : tang. AD : : rad. : co-ſine A (by theor. 1.) |
| 12 | All the three angles  A, B, and ACB | A side, ſuppoſe  AC | As co-tang. (ABC + A)/2 : tang. (ABC — A)/2 :: tang. ACB/2 : tan. of the angle included by the perpendicular and a line bisecting the vertical angles ; whence ACD is alſo known : then (by theorem 5.) tang. A : co-tang. ACD : : rad. co-ſine AC. |

The following propositions and remarks, concerning ſpherical triangles (ſelected and communicated to Dr Hutton by the reverend Nevil Maſkelyne, D. D. Aſtronomer Royal, F. R. S.), will alſo render the calculation of them perſpicuous, and free from ambiguity.

1. A ſpherical triangle is equilateral, iſoſcelar, or ſcalene, according as it has its three angles all equal, or two of them equal, or all three unequal ; and *vice verſa.*

2. The greateſt side is always oppoſite the greateſt angle, and the ſmalleſt side oppoſite the ſmalleſt angle.

3. Any two ſides taken together are greater than the third.

4. If the three angles are all acute, or all right, or all obtuſe ; the three ſides will be, accordingly, all leſs than 90⁰, or equal to 90⁰, or greater than 90⁰ ; and *vice verſa.*

*5.* If from the three angles A, B, C, of a triangle ABC, as poles, there be described, upon the ſurface of the ſphere, three arches of a great circle DE, DF, FE, forming by their interſections a new ſpherical triangle DEF ; each side of the new triangle will be the ſupplement of the angle at its pole; and each angle of the same triangle will be the ſupplement of the side oppoſite to it in the triangle ABC.

*6.* In any triangle ABC, or *AbC,* right-angled in A, 1st, The angles at the hypothenuſe are always of the ſame kind