from C to *c.* This is quite trifling, when the hearer is at a diſtance. Yet we ſee that sounds may be heard at a very great diſtance, at the end of long, narrow, cylindrical, or priſmatical galleries, It is known that a voice may be diſtinctly heard at the diſtance of ſeveral hundred feet in the Roman aqueducts, whoſe ſides are perfectly ſtraight and ſmooth, being plaſtered with ſtucco. The ſmooth ſurface of the ſtill water greatly contributes to this effect. Cylindrical or priſ­matical trumpets muſt therefore be rejected.

Let the trumpet be a cone BCA (fig. 2.), of which CN is the axis, DK a line perpendicular to the axis, and DFHI the path of a reflected sound in the plane of the axis. The laſt angle of reflection IHA is equal to the laſt angle of in­cidence FHC. The angle BFH, or its equal CFD, is equal to the angles FHD and FCH ; that is, the angle of incidence CFD exceeds the next angle of incidence FHC by the angle FCD ; that is, by the angle of the cone. In like manner, FDH exceeds CFD by the ſame angle FCD. Thus every ſucceeding angle, either of incidence or reflec­tion, exceeds the next by the angle of the cone. Call the angle of the cone *a,* and let *b* be the firſt angle of incidence PDC. The ſecond, or DFC, is *b — a.* The third, or FHC, is *b —* 2a, &c. : and the nth angle of incidence or reflection is *b — n a,* after *n* reflections. Since the angle diminiſhes by equal quantities at each ſubſequent reflection, it is plain, that whatever be the firſt angle of incidence, it may be exhauſted by this diminution; namely, when n times *a* exceeds or is equal to *b.* Therefore to know how many re­flections of a sound, whoſe firſt incidence has the inclination *b,* can be made in an infinitely extended cone, whoſe angle is *a,* divide *b* by *a* ; the quotient will give the number *n* of rcflections, and the remainder, if any, will be the laſt angle of incidence or reflection leſs than *a.* It is very plain, that when an angle of reflection IHA is equal to or leſs than the angle BCA of the cone, the reflected line HI will no more meet with the other side CB of the cone.

We may here obſerve, that the greateſt angle of incidence is a right angle, or 90⁰. This found would be reflected back in the same line, and would be incident on the oppo­ſite side in an angle = 90⁰ — *a, &c.*

Thus we ſee that a conical trumpet is well ſuited for con­fining the sound : for by prolonging it ſufficiently, we can keep the lines of reflected sound wholly within the cone. And when it is not carried to ſuch a length as to do this, when it allows the sounding line GH, for example, to eſcape without farther reflection, the divergency from the axis is leſs than the laſt angle of reflection BGH by half the angle BCA of the cone. Let us ſee what is the connection be­tween the length and the angle of ultimate reflection.

We have sin. *b — a* : ſin. *b —* CD : CF, and CF = CD × (sin. *b)/(sin. b — 2a),* and sin. *b — 2a :* sin. *b — a =* CF : CH, and CH = CF × (ſin. *b — a)/(*sin. *b)/(*ſin. *b—a)* × (sin. b — *2a),* = CD× (sin. b)/(sin. *b—2a), &c.*

Therefore if we ſuppoſe X to be the length which will ſin. *b* give us *n* reflections, we ſhall have X = CD × (sin. b)/(sin. b — na).

Hence we ſee that the length increaſes as the angle *b — na* diminiſhes ; but is not infinite, unleſs *n a* is equal to *b.* In this caſe, the immediately preceding angle of reflection muſt be *a,* becauſe theſe angles have the common difference *a.* Therefore the laſt reflected sound was moving parallel to the oppoſite side of the cone, and cannot again meet it. But though we cannot aſſign the length which will give the nth

reflection, we can give the length which will give the one immediately preceding, whoſe angle with the ſide of the cone is *a.* Let Y be this length. We have Y = CD × (sin. *b)/sin. a).* This length will allow every line of sound to be **re­**flected as often, ſaving once, as if the tube were infinitely long. For suppoſe a sonorous line to be traced backwards, as if a found entered the tube in the direction *ih,* and were reflected in the points *h, f, d, δ,* D, the angles will be continually augmented by the constant angle *a.* But this augmentation can never go farther than 90⁰ + 1/2a. For if it reaches that value at D, for inſtance, the reflected line DK will be perpendicular to the axis CN ; and the angle ADK will be equal to the angle DKB, and the sound will come out again. This remark is of importance on another **ac­**count.

Now ſuppoſe the cone to be cut off at D by a plane per­pendicular to the axis, KD will be the diameter of its mouth piece ; and if we ſuppoſe a mouth completely occu­pying this circle, and every point of the circle to be ſono­rous, the reflected sounds will proceed from it in the ſame manner as light would from a flame which completely occu­pies its area, and is reflected by the inside of the cone. The angle FDA will have the greateſt poſſible sine when it is a right angle, and it never can be greater than ADK, which is — 90 + 1/2a*.* And ſince between 90⁰ + 1/2a, and 90 — 1/2*a,* there muſt fall ſome multiple of a; call this multiple *b.* Then, in order that every sound may be reflected as often as possible, ſaving once, we muſt make the length of it X = CD × (S, b)/(S, a).

Now ſince the angle of the cone is never made very great, never exceeding 10 or 12 degrees, *b* can never differ from 90 above a degree or two, and its sine cannot differ much from unity. Therefore X will be very nearly equal to CD/(S, a) which is alſo very nearly equal to CD/(2S, 1/2a); becauſe *a* is ſmall, and the sines of ſmall arches are nearly equal and pro­portional to the arches themſelves. There is even a ſmall compenſation of errors in this formula. For as the sine of 90⁰ is somewhat too large, which would give X too great, 2 S, 1/2*a* is alſo larger than the sine of *a.* Thus let *a* be 12⁰ : then the neareſt multiple of *a* is 84 or 96⁰ both of which are as far removed as poſſible from 90⁰, and the error is as great as poſſible, and is nearly 1/160th of the whole.

This approximation gives us a very ſimple conſtruction. Let CM be the required length of the trumpet, and draw ML perpendicular to the axis in 0. It is evident that S, MCO : rad. = MO : CM, and CM ; or X = MO/(S, 1/2a), = LM/(2S, 1/2a), but = CD/(2S, 1/2a), and therefore LM is equal to CD.

If therefore the cone be of ſuch a length, that its diame­ter at the mouth is equal to the length of the part cut off, every line of sound will have at leaſt as many reflections, ſave one, as if the cone were infinitely long ; and the laſt reflected line will either be parallel to the oppoſite side of the cone, or lie nearer the axis than this parallel ; conſequently such a cone will confine all the reflected sounds within a cone whoſe angle is 2*a,* and will augment the sound in the proportion of the ſpherical baſe of this cone to a complete hemiſpherical ſurface. Deſcribe the circle DKT round C, and making DT an arch of 90, draw the chord DT. Then ſince the circles deſcribed with the radii