were numbers of pounds, m/n or *q* is a common number. And

because C and S are surfaces, or quantities of one kind, is also a common number.

This is the simplest expression that we can think of for the velocity acquired by the ship, though il; must be aknow- ledged to be too complex to be of very prompt use. Its complication arises from the necessity of introducing the leeway *x.* This affects the whole of the denominator ; for

the surface C depends on it, because C is = √A2 + B,, and A and B are analogous to A' cos. *3x* and B' sin *3x.*

But we can deduce some important consequences from this theorem.

While the surface S of the sail actually filled by the wind remains the same, and the angle DCB, which in future we shall call the trim of the sails, also remains the same, both the leeway *x* and the substituted surface C remains the same. The denominator is therefore constant ; and the ve­locity of the ship is proportional to √S ∙ V ∙ sin. *a ;* that is, directly as the velocity of the wind, directly as the absolute inclination of the wind to the yard, and directly as the square root of the surface of the sails.

We also learn from the construction of the figure, that FG parallel to the yard cuts CE in a given ratio. For CF is in a constant ratio to *Eg,* as has been just now demon­strated. And the angle DCF is constant. Therefore CF∙ sin. ft, or FH or *Gg,* is proportional to *Eg,* and OC to EC, or EC is cut in one proportion, whatever may be the angle ECD, so long as the angle DCF is constant.

We also see that it is very possible for the velocity of the ship on an oblique course to exceed that of the wind. This

will be the case when the number — ~~s~~uua ex.

*√ q* g- + sin.ft + **i**

ceeds unity, or when sin. α is greater than¼∕Ç-S- + sin. ft+*x.*

**Ö**

Now this may easily be by sufficiently enlarging S and di­minishing *b + x*. It is indeed frequently seen in fine sailers with all their sails set and not hauled too near the wind.

We remarked above that the angle of leeway *x* affects the whole denominator of the fraction which expresses the velocity. Let it be observed that the angle ICL is the com­plement of LCD, or of ft. Therefore, CL : LI, or A : B=I tan. ICL=I : cot. *b*, and B=A ∙ cotan. b. Now A is equi­valent to A' ∙ cos. *3x,* and thus ft becomes a function of *x.*

C is evidently so, being <∖Za2 -(- B2. Therefore before the value of this fraction can be obtained, we must be able to compute, by our knowledge of the form of the ship, the value of A for every angle *x* of leeway. This can be done only by resolving her bows into a great number of elemen­tary planes, and computing the impulses on each and add­ing them into one sum. The computation is of immense labour, as may be seen by one example given by Bouguer. When the leeway is but small, not exceeding ten degrees, the substitution of the rectangular prism of one determined form is abundantly exact for all leeways contained within this limit ; and we shall soon see reason for being content­ed with this approximation. We may now make use of the formula expressing the velocity for solving the chief prob­lems in this part of the seaman’s task.

And first let it be required to determine the best posi­tion of the sail for standing on a given course *ab*, when CE the direction and velocity of the wind, and its angle with the course WCF, are given. This problem has exercised the talents of the mathematicians ever since the days of Newton. In the article Pneumatics we gave the solution

of one very nearly related to it, namely, to determine the position of the sail which would produce the greatest im­pulse in the direction of the course. The solution was to place the yard CD in such a position that the tangent of the angle FCD may be one half of the tangent of the angle DCW. This will indeed be the best position of the sail for beginning the motion ; but as soon as the ship begins to move in the direction CF, the effective impulse of the wind is diminished, and also its inclination to the sail. The angle DC*w* diminishes continually as the ship accelerates; for CF is now accompanied by its equal eE, and by an angle ECe, or WC*w*. CF increases, and the impulse on the sail diminishes, till an equilibrium obtains between the resist­ance of the water and the impulse of the wand. The im­pulse is now measured by CE2 × sin.2eCD instead of CE2 × sin. 2ECD, that is, by EG2 instead of Eg2.

This introduction of the relative motion of the wind ren­ders the actual solution of the problem extremely difficult. It is very easily expressed geometrically : Divide the angle wCF in such a manner that the tangent of DCF may be half of the tangent of DCιe, and the problem may be con­structed geometrically as follows.

Let WCF (fig. 7∙) be the angle between the sail and course. Round the centre C describe the circle WDFY ; produce WC to Q, so that CQ=iWC, and draw QY paral­lel to CF cutting the circle in Y ; bisect the arch WY in D, and draw DC. DC is the proper position of the yard.

Draw the cord WY, cutting CD in V and CF in T; draw the tangent PD, cutting CF in S and CY in R.

It is evident that WY, PR, are both perpendicular to CD, and are bisected in V and D ; therefore (bv reason of the parallels QY, CF) 4 : 3=QW : CW,=YW : TW, RP : SP. Therefore

PD : PS=2 : 3, and PD:DS=2:l. Q.E.D.

But this division cannot be made to the best ad­vantage till the ship has attained its greatest velocity, and the an­gle *w* CF has been pro­duced.

We must consider all the three angles, *a*, *b*, and *x,* as variable in the equa­tion which expresses the value of V, and we must make the fluxion of this equation=0; then, by means of the equation B=A∙ contan. *b*, we must obtain the value of ft and of *b* in terms of *x* and x. With respect to *a,* observe, that if we make the angle WCF=*p*, we have *p=a+b+x∙,* and *p* being a constant quantity, we have a+b+x=0∙ Substituting for *a,* ft, a, and *b*, their values in terms of *x* and a?, in the fluxionary equation=0, we rea­dily obtain *x,* and then *a* and 6, which solves the problem.

Let it be required, in the next place, to determine the course and the trim of the sails most proper for plying to windward.

In fig. 6, draw FP perpendicular to WC. CF is the motion of the ship ; but it is only by the motion PC that she gains to windward. Now CP is=CF × cosin. WCF, or *V* cosin. (a+b+x). This must be rendered a maxi­mum, as follows.

By means of the equation which expresses the value of *v* and the equation B=A∙ cotan. *b*, we exterminate the quantities *v* and ft ; we then take the fluxion of the quantity into which the expression *ν∙*cos. (a+b+x) is changed by this operation. Making this fluxion=0, we get the equa­tion which must solve the problem. This equation will contain the two variable quantities *a* and *x* with their flux-