a small surface in comparison of the sails, must be very great: For the impulse of water is many hundred times greater than that of the wind ; and the arm *qG* of the lever, by which it acts, is incomparably greater than that by which any of the impulsions on the sails produces its effect ; ac­cordingly the ship yields much more rapidly to its action than she does to the lateral impulse of a sail.

Observe here, that if G were a fixed or supported axis, it would be the same thing whether the absolute force D*d* of the rudder acts in the direction D*d*, or its transverse parts D*e* acts in the direction De, both would produce the same rotation ; but it is not so in a free body. The force D*d* both tends to retard the ship’s motion and to produce a ro­tation : It retards it as much as if the same force D*d* had been immediately applied to the centre. And thus the real motion of the ship is compounded of a motion of the centre in a direction parallel to D*d*, and of a motion round the centre. These two constitute the motion round S.

As the effects of the action of the rudder are both more remarkable and somewhat more simple than those of the sails, we shall employ them as an example of the mechanism of the motions of conversion in general ; and as we must content ourselves, in a work like this, with what is very ge­neral, we shall simplify the investigation by attending only to the motion of conversion. We can get an accurate no­tion of the whole motion, if wanted for any purpose, by com­bining the progressive or retrograde motion parallel to D*d* with the motion of rotation which we are about to deter­mine.

In this case, then, we observe, in the first place, that the

angular velocity (see Rotation, No. 22.) is > an^>

as was shewn in that article, this velocity of rotation in­creases in the proportion of the time of the forces’ uniform action, and the rotation would be uniformly accelerated if the forces did really act uniformly. This, however, cannot be the case, because, by the ship’s change of position and change of progressive velocity, the direction and intensity of the impelling force is continually changing. But if two ships are performing similar evolutions, it is obvious that the changes of force are similar in similar parts of the evo­lution. Therefore the consideration of the momentary evo­lution is sufficient for enabling us to compare the motions of ships actuated by similar forces, which is all we have in view at present. The velocity *v,* generated in any time *t* by the continuance of an invariable momentary accelera­tion, (which is all that we mean by saying that it is pro­duced by the action of a constant accelerating force), is as the acceleration and the time jointly. Now what we call the *angular velocity* is nothing but'this momentary accele­ration. Therefore the velocity *v* generated in the time *t*

∕Pr

The expression of the angular velocity is also the expres­sion of the velocity *v* of a point situated at the distance 1 from the axis G.

Let *z* be the space or arch of revolution described in the time *t* by this point, whose distance from G is =1.

• F \* g(t

Thenz=t>i= -y⅛- < <, and taking the fluent *z =* F \* çO

*~~~j∙'p~~~~r~~~~Γ~~ e∙* This arch measures the whole angle of rotation

accomplished in the time *t.* These are therefore as the squares of the times from the beginning of the rotation.

Those evolutions are equal which are measured by equal arches. Thus two motions of 45 degrees each are equal.

Therefore because *z* is the same in both, the quantity F ∙ <∕Cτ

*~~~jip~~~~τ~~~~i~~* \*s a constant quantity, and *t*2 is reciprocally pro­portional to ’ θr \*8 proportional to J⅛l ,and *t* is proportional to —*Jl'r~.* That is to say, the times of the βi- √F∙5-G

milar evolutions of two ships are as the square root of the momentum of inertia directly, and as the square root of the momentum of the rudder or sail inversely. This will enable us to make the comparison easily. Let us suppose the ships perfectly similar in form and rigging, and to differ only in length L and *l; J"ι ∙* Rs is to ~∕'*pri* as Ls to *l5.* For the similar particles P and *p* contain quantities of matter which are as the cubes of their lineal dimensions, that is, as L3 to *P.* And because the particles are similarly situated, R2 is to r2 as L2 to *l2.* Therefore P ∙ R2 : *p* ∙ r2=L5 *l*5. Now F is to *f* as L2 to *P.* For the surfa<es of the similar rudders or sails are as the squares of their lineal dimensions, that is, as L2 to *P.* And, lastly, *Gq* is to *gq* as L to *l,* and there­fore F ∙ *Gq ∙.f∙gq=L3 : l3.* Therefore we have T2 : t2 =

F∙Gy *f∙gq* L3 *P*

Therefore the times of performing similar evolutions with similar ships are proportional to the lengths of the ships when both are sailing equally fast ; and since the evolutions are similar, and the forces vary similarly in their different parts, what is here demonstrated of the smallest incipient evolutions is true of the whole. They therefore not only describe equal angles of revolution, but also similar curves.

A small ship, therefore, works in less time and in less room than a great ship, and this in the proportion of its length. This is a great advantage in all cases, particularly in wearing, in order to sail on the other tack close-hauled. In this case she will always be to windward and a-head of the large ship, when both are got on the other tack. It would appear at first sight that the large ship will have the advan­tage in tacking. Indeed the large ship is farther to wind­ward when again trimmed on the other tack than the small ship when she is just trimmed on the other tack. But this happened before the large ship had completed her evolution, and the small ship, in the mean time, has been going for­ward on the other tack, and going to windward. She will therefore be before the large ship’s beam, and perhaps as far to windward.

We have seen that the velocity of rotation is proportional, *cateris paribus,* to F × *Gq.* F means the absolute impulse on the rudder or sail, and is always perpendicular to its sur­face. This absolute impulse on a sail depends on the obli­quity of the wind to its surface. The usual theory says, that it is as the square of the sine of incidence ; but we find this not true. We must content ourselves with expressing it by some as yet unknown function *φ* of the angle of incidence *a,* and call it *φa,∙* and if S be the surface of the sail, and V the velocity of the wind, the absolute impulse is nV2S × *φa∙* This acts (in the case of the mizen-topsail, fig. 10.) by the lever *qG,* which is equal to DG × cos. DGg, and DGq is equal to the angle of the yard and keel ; which angle we for­merly called *b.* Therefore its energy in producing a rota­tion is nV2S × *φa* × DG × cos. *b.* Leaving out the constant quantities *n,* V2, S, and DG, its energy is proportional to *φa* × cos. *b.* In order, therefore, that any sail may have the greatest power to produce a rotation round G, it must be so trimmed that *φa* X cos. *b* may be a maximum. Thus, if we would trim the sails on the foremast, so as to pay the ship off from the wand right a-head with the greatest effect, and