ber that denotes the place or order of that term in the se­ries. In conformity to this method, putting the symbols T<∣), T<j⅛ T(s), &c^ to denote the terms of any series whatever, we may express it generally thus,—

T<i) + T(j) + T[j) ∙ ∙ ■ + T(∙), where the characters (1), (2), are meant to denote the place or order of the terms to which they are joined (the first term being supposed to have the place 1, the second term the place 2, and so on), and (*v*) is put for any indefinite number.

The nature of the arithmetical series

*a* + (β + *d) + (a + 2d)* + (β + 3d) +, &c.

will be defined by the equation

T(.) = « + («,\_*l)d;*

and, in like manner, the nature of the geometrical series *a + ar + ar, + αr3* +, &c.

will be expressed by the equation T(.) = βr'~1.

(7.) As the expression for the value of the indefinite term T<∙> becomes identical with all the terms of the se­ries in succession, by substituting the numbers 1, 2, 3, &c. one after another, for *v,* that expression is called the *general term* of the series. In the series

*ab ab ab*

α + b + ⅛∑∑i+ 3^≡26+⅛∑∑3δ +’

the general term is evidently 7 τr—*--z* -τ-r∙

o (o— l)α—*{v—2)b*

(8.) We shall investigate the sum of any number of terms of such series as have their general terms expressed by any one of the following algebraic functions.

*\_ v(v+*1) t>(e+l)(t> + 2) n(t>+l)(e+2)(e + 3)

”· —Γ2~, —p∏—’ m ’&c·

Problem I. It is proposed to find the sum of *n* terms of the series of which the general term is the first function.

By putting 1, 2, 3, &c. to *η* successively for *v,* it appears that the series to be summed is

l+2 + 3 + 4∙∙∙+n.

Now, as *V = ~~v~~~~^~~~~,~~ ~~f~~* \*e h°ve> putting

in this formula 1, 2, 3, ∙ ∙ ∙ to *n* successively, for *v*,

' , I’» λ

2-\*48 L·?

2 2 ’

3∙4 2∙3

2 2 ’

. 4∙5 3-4 \* = -2^ Γ’

.. ,\_(»—·)» (Λ —2)(n-1)

\_ \_ "(” + Ό \_ (» —1)"

2 ∙2 "

Let the sum of the quantities on each side of the sign = be now taken ; then, observing that each of the frac­tions on the right-hand side, with the exception of ⅛-,

occurs twice, once with the sign +, and again with the sign —, by which it happens that their aggregate is = 0, it **is** evident that we have

l + 2 + 3 + 4∙∙∙+η = -∖√2-<

Prob. II. It is proposed to sum *n* terms of the series having for its general term the second function

p(p + l)

1 ∙ 2 -

This series, by substituting 1, 2, 3, &c. successively for *v,* is found to be

1 ∙ 2 2 ■ 3 3 ∙ 4 \_ n(n -f- 1)

P^2 + Γτ2 + Γr8 ’ ’ + 1 ∙ 2 - We now, following the mode of proceeding employed in last problem, put the expression —y-⅛-under this form,

e(e + l)(t> + 2) (n— l)t,(v + 1)

1 ∙ 2∙3 1 ∙2∙S ’

to which it is evidently equivalent, and, substituting 1, 2, 3, &c. successively for *υ,* find

1 ■ 2 1 ∙ 2 ∙ 3 n

1 ∙ 2 ~ 1 ∙ 2 ∙ 3 ’

2∙S2∙3∙4 l∙2∙3

1∙2~1∙2∙3-Ί ∙2∙3,

3∙4 3∙4∙5 2∙3∙4

I∙2-l∙2∙3l∙2∙3,

4 ∙ 6 \_4·ό· 6 3∙4∙5

l∙2~l∙2∙S l∙2∙3

»(»+ 1) \_ »(« +»)(” + a) (n—l)∏(n+ 1 )

1∙2 - 1∙2∙3 l∙2∙3 ,

In this problem, as in the former, it appears that each quantity on the right side of the equations, except *n (n* + 1) *(n* + 2)

~~—1\*~~> ∙~~ **~~3 ‘~~**—^, occur8 twιce' an<\* wlth contrary signs ;

therefore, taking the aggregate of the terms on each side, we have

1 ∙2 ι 2∙3 3∙4 ι 4∙5 l «(» + 1)

l∙2 + l∙2+l∙2 + l∙2 " + l∙2

\_«(» + \*)(” + ¾)

- 1 ∙2∙3

(9.) It will be obvious, by a little attention to the solu­tions of these two problems, that in each, the terms of the series to be summed are the differences betwixt the adja­cent terms of another series, namely, that which has for its general term the function next in order to the general term of the series under consideration ; that is, the terms of the series whose general term is *v*, are the differences betwixt those of the series having ~~— γ y^>~~ for its general term ; and again, the terms of this last are the differences of the terms of the series having +\_\_++\_ for its general term. Now, as the sum of the differences of any series of quantities whatever which begins with θ must necessa­rily be the lost term of that series,@@1 it follows that the sum of all the terms of each of the series we have considered must be equal to the last term of the next following series ; and this term is necessarily the expression formed by sub­stituting *n* for *v* in its general term, that is, the sum of the series 1 + 2 + 3 ∙ ∙ ∙ -(- n, which has *v* for its general

. n(jt 4- I) , , .,

term, is **~~’ ∣ . g~~ ~~z~~** ! and the sum of the series

**@@@1 For example, let the quantities be 0, α, 6, e, *d ;* then it is manifest that (α — 0) + (i∣ — a) + (e — ⅛) + (4 — c) w *d.***