m-,i,ι5√l"÷l-i-÷ill-÷ιl

\_„(„ + i)<,. + 2) + ÜÜ±-!>,

and, by proper reduction,

S = 13 + 23 + 33 + 43∙∙∙ + wj=--⅞2~~)ζ~~ 4»

Corollary. We have found (Prob. I.) that 1 + 2 + 3 + 4∙∙ + n=

therefore, comparing this with the result just now obtained, it is evident that

(1 + 2 + 3 + 4∙∙∙ + n)'= 1’+ 23 + 33 + 43∙∙∙ + w3. This is a very curious and elegant property of numbers.

(11.) It is manifest that, by the mode of proceeding em­ployed in last problem, we may investigate the sum of *n* terms of the series

1™ + 2m + 3n, + 4ra +, &c.

*m* being any whole positive number whatever ; and indeed in the very same way we may find the sum of any number of terms of a series whose general term is

*a + bυ + cvi + dυs* +, &c.

where *a* and t, &c. denote given numbers ; namely, by transforming it into a function of the form

A + B *υ* + C *υ (v* + 1) + Dr(» + 1) (» + 2) +, &c. where A, B, and C, &c· denote constant quantities. Our limits, however, will not allow us to go into particulars.

(12.) The next class of series we shall consider, compre­hends such as may be formed by the successive substitu­tion of α, o+ 1, *α+* 2, &c. (a being put for any given quantity whatever) in the βeries of functions

1 1 1 .

*V (v* + 1 ), *v* (v + 1 ) (» + 2), *v* (v + 1 ) (« + 2) *(υ* + 3), C’ We shall begin with the first of these.

Prob. V. It is proposed to find the sum of n terms of the series

α(β+ 1) + **(o+l)(o +** 2) + (α + 2)(α + 3)+, &C\* which is formed by substituting a, a + 1, β + 2, &c. suc­cessively for *v* in the general term , ∖ ,,.

*v(υ* + 1)

Whatever be the value of *v,* we have 1 \_l 1

**e(v +** I)" *v v* + 1’

therefore, proceding as in the foregoing problems, we get 1 \_ 1 1

*a(a* +1) *α* β+ l’

1 \_ 1 1 (β + 1) *(a* + 2) - *a* + 1 *a* + 2’

1 I 1

(o + 2) (o + 3) - *a* + 2 *a* + 3’

\_1 1 1

- («+ «— 2 ) (o + n—I) o + »— 2 α+n—1’

1 \_ 1 l\_

(a + n—l)(o + n) o + n—l *a + η*

Here it is evident that the terms of the series to be summed are the differences betwixt every two adjoining terms of this other series,

-1+∙∙ 1 +' 1 p 1 ∣ 1 ∙

*a* α + 1 o + 2 α + 3 *a + » ’*

hence it immediately follows, that the sum of all the terms

of the former is the difference between the two extreme terms of the latter ; that is,

- 1 - + 1 .... + ∖ α(α+1) (α+l)(α + 2) *(a+n —* l)(a+n)

\_ 1 \_l\_

*a a + n*

If we suppose the series to be continued *ad infinitum,* then, as *n* will be indefinitely great, and - indefinitely

small, the sum will be simply - ; or, in other words, the

α

fraction - is a limit to the sum of the series. *a*

Prob. VI. Let it be required to find the sum of *n* terms of this series,

1 ι 1 o(o+l)(o + 2) + (o+ l)(o + 2)(α + 3)

+ (a+ 2) (o + 3) (β + 4) +, &C·5

the general term in this case being —-,—, \_. ,—i—i∖.

*v(υ* + 1) *(v* + 2√

2 11

Because ———— = —therefore, multiplying

*υ{y* + 2) *v v + 2 ∣ j b*

~~by~~ ~~¾⅜⅞i)~~~~, we have~~

**t>(o+ l)(t> +** 2) = ÷ {*v∖υ* + 1)-(„+ l)(p + 2)} i and hence, by substituting α, a + I, o + 2, &c. successively for *v,*

a(a+l)(o + 2) 2{a(o+l) (o+l)(a + 2)∫ ’

(o + l)(β + 2)(o + 3) = 2 { (a+ l)(o + 8)

~(o + 2)(o + 3)},

**1 -■/ 1**

(o + 2)(o + 3)(o + 4) 4(a + 2)(a + 3)

~(a + 3) (a + 4)},

1

(o + n — 1) (a + n) (a + n + 1)

- 2 { (a + n — 1) (a + n) *(a* + n) (a + *n* + 1) ∫ ' Hence it appears that the terms of the series to be summed are the halves of the differences of the terms of the series *a(α* + J) (o + 1) (« + 2) τ (o + 2)(β + 3) ’ ’

-I- 1

fo + n)(a + n+ 1)’

consequently, the sum of all the terms of the former is half the difference between the extreme terms of the latter, or is =

*2 { a* (α + 1) - (a + n) (a + n +1) } '

(13.) From these two particular cases it is easy to see how we may sum the series when the general term is

1

*v (v* **+ 1) (» + 2) . . .** *(υ + p)'*