*p* being any whole number whatever : for since

*P* 1 1\_

p(p + *p) v V + p*

therefore, multiplying the denominators by all the factors which are intermediate between *v* and *ν + p,* we have

*v* (r + I)? *(v'+*2) "',. (t> + *p)*

1

*- v (v* + 1) *(v* + 2) ... (p + *p —* 1)

1

(p+l)(p + β)(e + S)...(e+p)∙

Now the latter side of this equation is a general expression for the difference between any two adjacent terms of a series whose general term is

\_\_2 **1**

**r(r+** 1)(p + 2)...(p +/> —1), therefore the difference between the first and last terms of this series must be the sum of the series whose general term

is the function on the other side of the equation, viz.

*. . ' , P*

® (® + 1) (p + 2) . . . (e + *pf* Hence we have the following very general theorem.

THEorem. *Let* a *denote any number whatever, and let* l, 2, 3 . . . p 6e α *series of numbers, each of which exceeds that before it by unity ; the sum of* n *terms of a series formed by substituting the numbers* a, a +1, a + 2, &c. to a + n — 1 *successively for* v *in the function*

1

p *(v* + 1) *(y* + 2) . . . (p + *p)*

*is equal to*

1 j α(<t + 1) (α+^i2) ... (α + j>-r-l)

Ü) 1

' (a + n)(o + n+l)(o+a + 2).. . + (o + **n+∕‰-i)^**

Corollary. *The same series continued* ad infinitum *is equal to*

1 1

*p ' α (a* +1) (α + 2) ... *(a + p—* 1)"

(14.) We shall now give examples of the application of this theorem.

*Example* 1. Required the sum of *n* terms of the series I 1 1

*2 ∙ 3 ∙ 4. ∙ 5 ’ 3 ∙ ⅛ ∙ 5 ∙ 6 ' 4L ∙ 5 ∙ 6 ∙ 7 ,*

The terms of this series are evidently produced by the suc­cessive substitution of the numbers 2 3, 4, 5, &c. for r in the function

1

p(p+ l)(p + 2) (p + 3)’

therefore, comparing this expression with the general for­mula, we hâve *a* = z, *p —* 3, and the sum required

= IÎ\_J ~ 1, )

tl2∙3∙4 (2 + ») (3 + n)(4 + n)∕∙

*Ex.* 2. Required the sum of the series

1 -l , . 1 . 1 t\_.~

I ∙ 4 ∙ 7 ' 4 ∙ 7 ∙ 10. 7 ∙ 10 ∙ 13 ^t" 1,0 ∙ 13 ∙ 16 ^\*^,

continued αrf *infinitum.*

By a little attention it will appear that its terms are pro­duced by the substitution of the numbers ]∙, 1+ 2⅛, &c. successively for p in the function

1 \_ 1 3e (3p + 3) (3p +^6) - 27v(p+ 1) (e + 2)‘

In this case, *a — ⅜, p* = 2, therefore the sura is

**I×l× \_L\_-1**

2 27 ⅜ X 1⅜ - 24'

(15.) When the function from which the series is derived has not the very form required in the theorem, it may bc brought to that form by employing suitable transformations, as in the two following examples.

*Ex.* 3. It is proposed to find the sum of the series

±+±+-i+±+ &c

1∙4 2∙5 3∙6 4∙7 +,

continued *ad infinitum.*

This series is evidently formed by the substitution of the numbers 1, **2,** 3, &c. successively for *v* in the function ~~t~~~~, (~~~~t~~~~, 4- ¾y~~ This expression, however, does not in its pre­sent form agree with the general formula, because the fac­tore **p + 1,** p + 2 are wanting; therefore, to transform it, we multiply its numerator and denominator by *(v* **+ 1) (p + 2),** and it becomes

(p ÷ I)(p + 2) p(p+ 9 (p+2)(p+3)∙

We next assume its numerator

(p + l)(e + 2) = A(e+ 2)(p + 3) + B(c+ 3) + C, and by multiplying get

p! **+ 3e + 2 = At? + (5A + B)** *v* **+ (6A + 3B + C);** therefore, that **p** may be indeterminate, we must make

**A = 1, 5A +** B = 3, **6A +** 3B + **C =** 2, from which equations we get *A* = 1, B — 3 — 5A ≈ — 2, **C =** 2- **6A —** 3B = 2, so that

\_J (P + 2) (p + 3) - 2 (P + 3) + 2

p(p+3) p(e + 1) (p + 2}p + 3)

\_ 1 2\_\_\_\_

**p (p + I)** *v* (p + 1) (p + 2)

+ ?

' p(p + 1)(p + 2)(p + 3)'

Thus it appears that the proposed aeries is resolvable into three others, the general terms of which all agree with the theorem- Now the sum of the infinite series whose general

term is . ∖—r- appears by tħc theorem to be -, orl,be- **p(p + I) r, z** *a*

cause α = 1, and the sum of the infinite series whose ge-

-r2

neral term is —;—, ,. .—r-5s is in like manner found **p(p +** 1>(p **+ 8)**

t0 be ΐ + Γ⅛ = ΐ; and> ,astl>' the infinite series 2

whose general term is . , ,.-.—-,-⅞⅛--.—, o. is

e p (p + l) (e + 2) (p + 3)

\* 2 1 1

~~— j , g . g~~ “§· therefore, collecting these into one, the

sum of the proposed series is 1 — 1 + **5 = TΞ>** the answer.

*Ex.* 4. Required the sum of the infinite series 1 . 2 3 , 4 1 s

2 ■ 3 ∙ 4 + 3 ∙ 4 ∙ 5 + 4 ∙ δ ∙ 6 ^,^ 5 ∙ 6 ∙ 7 +, &C·

The terms of this series are evidently formed by the sub­

stitution of the numbers 2, 3, 4, successively in the func­tion

p—I

p(p + 1) (o + 2)’

Now p — 1 = r + 2 — 3; therefore

p \_ 1 I I S i 1

**p(p+ l)(p + 2)~β(s>+ 1>** p(p+1>(p+2)∙

Thus it appears that the proposed series is reducible to two