others, one having its termä produced by the substitution of

2, 3, &c. for *v* in the function —, and the other by e(υ + 1) j

3

a like substitution in the function —,—. .. .—r-Now, c(t' + l)(u+ 2)

by our theorem, the sum of the first of these is -, and that of 3 1 1

the second is —5- ' —5 = — -, therefore the sum of the *A A ' .s* **4**

. . . 1 1 \_ 1

proposed scries is 5 — - = -.

From these examples it is sufficiently evident how the theorem is to be applied in other cases ; and it appears also, that by means of it we can sum any series whatever whose general term is of the form

∣ 5. , C ,

t∙(l + e)^t^r(l+ u)(p+ l)^l^r(e + ∣)(e + 2)(υ +3) +, &c.

or admits of being reduced to that form.

(16.) It deserves to be remarked, that the series

1 1 I 1 1

Î + 2+3 + 4 +5+’&C· which is of a very simple form, and in appearance of the same nature as those we have summed, does not, however, admit of being treated in the same manner; and indeed if it be continued *cd infinitum,* its sum is infinite, that is, it exceeds any number which can be assigned. The truth of this assertion will be evident if we can show that a certain definite number of its terms, beginning with any proposed term, can always be found, the sum of which shall exceed an unit, of 1 ; for this being the case, as we can go on con­tinually in assigning such sets of terms, we can conceive as mnny to be taken as there are units in any proposed numbcr, however great ; and therefore their sum, and much more the sum of all the terms of the series, from its begin­ning to the end of the last sets of terms, will exceed that number. Now, that this can always be done may be proved as follows :

Let the term of the series from which we are to reckon bc^, then, if the thing he possible, and if *n* be the requi­site number of terms, we must have

1 I I 1\_ l

w+α-∣-l+α-f-2 a -f- 3 + π -∣- n— *1 '*

*Now,* because

0(1 + l)≈=0+2÷∙ √.+i)'=.+.+i+⅛

and in general,

α(l+l)i'=σ + p + ^=21+,ftc. therefore, *p* being any whole number,

*a* ( 1 + -) , *a + ρ,* and consequently

β⅛7¾ι

hence it follows, that the series

, 1 1 , 1 1 *a + a* + I + *a* -(- 2 ■" + α -j- n — I

w ill be greater than the other series

μ≠Γ≠Γ≠Γ

**1**

+ ∕. ∣∖i~l α (1 + 0)

Now, this last being evidently a geometrical series, of which the common ratio is ——r its sum is

■ +-ι,

1 4- 1 \_ -!-\_

' FT

therefore the sum of the series

l I 1 1 1 1

*a* α-f-l+α4-2 + α-}-3- +a-f-n — I

will always be greater than this expression ; but if we sup­pose *η* so great that the quantity (l -(- -) is equal to or

exceeds *a,* which is evidently always possible, then the above expression for the sum of the geometrical series will be equal to 1, or will exceed 1 ; therefore the same number of terms

of the series-^ + -——, -1 -—7: + —7—4-, &c. will al-

αlα-∣-l 1 *a* -{- 2 , *a* 4- 3

ways exceed 1 ; now this is the property of the series we proposed to demonstrate.

Whcn *a =* (l 4- ^) , then *dt = a* (l -f- ; but

this quantity is greater than *a -j- n—* 1, the denominator of the last term of the series

i + ^+LLa+-J-... + I—,

*a' a-j-L'a-j-2 a -j- 3 a* -j- *η —* 1 the sum of which, we have proved, will upon that hypothe­sis exceed unity ; much more then will the sum exceed unity if we suppose the series continued until the deno­minator of its last term be equal to or greater than a\*. Hence, beginning with the term ∣, it appears that

i + ⅜ + 4\_ ,>

, 1

i + i∙∙∙ +⅛τpΙ,

, 1

9⅛+ 2V∙∙∙ + 676 = 26\*^1-

K⅛ + 5⅛ ” ∙+ 458329 \_ 677. 1,

<S∙∙c.

Although the sum of the series we have been considering is infinite, yet it evidently increases very slowly ; indeed it is a limit to all such as have a finite sum ; for every in­finite series the terms of which decrease faster than the re­ciprocals of an arithmetical progression is always finite.

(17.) We have already explained what is meant by a *recurring series* (2.). We shall now treat briefly, first, of their origin ; next, of the way in which they may be sum­med ; and, lastly, of the manner of determining the *general term* of any particular series.

The series which is produced by the development of a rational algebraic fraction has always the property which constitutes the characteristic of the class called *recurring* (2.) ; and, on the other hand, any series having that property being proposed, an algebraic fraction may be found, by the expansion of which the series shall be produced.

1 \_L. 2 *X*

The fraction —— -, for example, by dividing the

numerator by the denominator, is convertcd into the infi­nite series

1 ψ3χ-j-4χSψ7x3-p 11 *X\** -f∙ 18 *xi* -∣-, &c.