which is of such a nature that if T, T', T", denote any ' three of its succeeding terms, their relation to one another Is expressed by the equation

T''=Tx, + T'a∙.

If we employ algebraic division to convert the fraction into a series, the law of its terms will not appear so readily as if we were to use the method of indeterminate co-effi­cients. By this method we assume the fraction equal to

A -f- B *X* 4- C *xt* 4. D*x3* + E*X\** -(-, &c. ; and hence, multiplying by the denominator, and bringing all the terms toone side, as explained in Algebra, § 159, we have

A + B1 +Ci + D)

— 1 — A J- *X—* B *x2-* C ’- x1 +, &c. = 0 ;

-2i — A) -B)

and hence

A —1-0, C —B —A = 0,

B—A — 2 = 0, D — C —B = 0,

&c.

From these equations it appears that the law of the series is such as we have assigned.

The equation expressing the relation which subsists among a certain number of succeeding terms of a recur­ring series is called its *scale of relation.* The same name is also sometimes given to the equation expressing the con­nection of the co-efficients of the terms. Thus the scale of relation of the foregoing series is either

T'=Tλ+ *Υ'x',*

where T, T', and T" denote any three succeeding terms of the series ; or it is

R = P + Q,

where P, Q, and R denote their numeral co-efficients.

(18.) We come next to show how the sum of any pro­posed number of terms of a recurring scries may be found. Let the series continued to *n* terms be

T(i> 4- 1 (»4· T∣∣)... 4 T <»—2) -)- T (»—i) + T (n), where the characters T <ιj, T <a,, &c. denote the succes­sive terms, and the numbers (1), (2), &c. their order or place ; and as, whatever number of terms is contained in the scale, the manner of summing the series is the same, we shall in what follows, for the sake of brevity, suppose that it consists of three, in which case it may be expressed thus,

*p'ι* (a-2) 4- *gΓ* (.-I) 4- rT (») = 0, where *p, q, r* denote certain given quantities.

The scale of relation affords the followring series of equa­tions,

pT (i) 4- *q* T (2) 4- rT <s) = 0, *pf* (2) + yT (3) -(- rT (4) = 0, pT (J) 4- yT (4) -J- rT (51 = 0,

pT (»—2) 4- *qT* (»—η 4- rT (») = 0.

Taking now the sum of these equations, we get

∕> {T (i) 4- T (2> 4- T (»>· ∙ ∙ 4- T (>-2)j I

*+ q* {T<s) 4- T(J) 4- T(4)∙ ∙∙ 4- T (»-!)} > = 0.

4-r {T«) 4- T(4) 4- T(β),∙∙ 4- T(.)} **J** But, putting s for the sum of *n* terms of the series, this equation may manifestly be expressed thus,

*P* {i—T (n) — T(β-ι,} )

+ 7 {s~ T(» — T ου} f=0∙

4-r {s-**T(∣)-T(2>} )**

Hence, after reduction, we find *s =*

p{T(.-∣) + τw} +y{T(l)4-T(,)} + r{T(∣) + Tθ)}.

*,. , . P + + τ*

from which it appcars that in this case the sum depends only on the first two and the last two terms of the series.

Example. It is proposed to find from this formula the sum of η terms of the scries

1 4- 2λ 4- 3λ, 4- 4∙r3 4\* 4-, &c.

its scale of relation being

λs T (» —2) — *2x* T (»—1) 4- T (∙) = 0.

Here *p* = x2, *q = — 2x, r* = I, therefore, observing that the last two terms of the series must be (n — 1) *xλ~1* and na,n-1, we have, after substituting and reducing,

\_ 1 — (n 4 l)ι" 4- x"\*l t ~ 1*1— 2ι* 4- *xl*

This formula will not apply in the case of *x =* 1, because then the numcrator and denominator are each = 0 : but in such cases as this we may find the value of the function which expresses the sum by what is delivered at § 55, **FLUX1ONS.**

(19.) The process by which we have determined the value of *n* terms of the series T (i) 4- T (2) 4- T (3) +, &c. will also apply to the finding the rational fraction from which the series may be deduced, which is also the sum of the series continued *ad inf nilum.* For in this case the equa­tion from which we have deduced the sum being

*P* {T (I) 4^ P W + A (3) 4-, &c. [■ j

+ y {T (2) 4- T (3> 4- T («) 4-, &c.} > = 0,

+ r {T (3) 4- T (4) 4- T (β) 4-, &c.} )

that is,

*ps* 4. y {s \_ T (I)} 4. r {» — T (i) — T w} = 0,

V {? + r} T (I) + r T (2)

we have *s = — r*

p + y + r

For example, let it be required to find the fraction which, being developed, produces the series

1 4^ *%x* + S∙rs + 4x3 4^> &C· the scale of relation of which is

*xi* T (∏-J) — 2∙rT (»—1) 4- T <») 4- 0.

Here *p* = «\*, y = — *2x, r* = 1, T (∣) = 1, T (2) = *2x ;* therefore, substituting in the formula, we get

1 l\_

1 — *2x* 4\* x" ( I — ∙t)'

for the fraction required, or for the sum of the series con­tinued *ad infinitum.*

(20.) We come now to the last branch of the theory of recurring series which we proposed to consider, namely, how to find in any case the *general term.*

We shall begin with the most simple, and suppose the fraction to be ~l , which being expounded into a series by division, is

*a* 4- *apχ* 4- *afil χ1* 4^ *αp3 χ3 ^∣^> &c·*

Here it is immediately manifest that the general term is σ∕>"-1 a·"—\*.

Next let us suppose the fraction to be ∣ Tαj√-^t3.τ∙-

Let the two roots of the quadratic equation 1 — *ax — βx'*

= 0 be *x = -, x = -,* so that 1 — *px —* 0, and 1 — *qx ρ q*

= 0 ; therefore 1 — *ax — βxl —* ( 1 — *px)* ( 1 — *qx)* ; thus we have

*a* 4- *hx \_ a* 4^ *bχ*

1 — *αx — βχ∙ ~~* (1 —∕xr)(l — *qχ)*

Let us assume this expression equal to

\_Q\_

1 — *px~ i. — qx'*

wherc P and Q denote quantities which are to be inde­pendent of *x ;* then, reducing to a common denominator, we have

\_ a ÷ kr \_ P + Q — (yP + pθ)5

(l-^x)(l-yr)-"^(Γ-px)(l-y.r)

Hence, that *x* may remain indeterminate, we roust make