P + Q = σ, çP + pQ = — *b,* and from these equations we get

*p \_ '1P + b* Q \_\_g9 + b *p-g, p-q'*

Now, by the operation of division, we find

~~i~~~~-~~~~Γ~~~~pj~~ *= p + ppχ* + ppiι\*' +■ 4c∙

-r-—— = Q + *Qqx* + Q∣jr⅛2 +, &c. ;

1 — *qj:*

*.< r ■ σ + bx* P , Q .. p 1

therefore, since τ τc^t = ·: 1- - , it lol-

1—*ax — pxi* 1—*px* 1 — *qx*

lows that the development of the fraction i—~∙^-⅛s-5,

1 — *ax — pxi*

which proceeds according to the powers of *x,* is

(P + Q) + (Pp + *Qq) x* + (Pp\* + Qr∕2) x2

+ (pp3 + Qr1) a"3 +> &c·

And here it is evident that the general term is (Pp"-1 + Qy"-') x"~l.

Let us take as a particular example the fraction

1 — *x*

1 *-x*—2xs' wh'ch when expanded into a series, becomes

1 + Ox + 2x’ + 2x5 + 6x, + 10xs + 22χβ + 42x7 + 861“ +, &c.

Here, from the equation 1 — *x — 2x"* = 0, we get *x = ⅜* and x = — 1, so that 1 —*2x* and 1 + *x* are divisors of the function 1 — *x —* 2x2, that is, 1 — *x — 2x, = (1* + x) (1 — *2x);* hencep = — 1, *q =* 2; and sinceα = 1,0 =— 1, therefore P = Q = ⅛, and the general term (Pp" —1 -Ι­

Ο/·-1) x"-1 becomes, by substituting,

r2 11 2\*—1-4-9

{5(-1)-> + 32.-}x-,= ^⅛-a-,,

where the sign + is to be takcn when *n* is an odd number ; but the sign — when *n* is even.

Sometimes the values of p and *q* will come out imaginary quantities ; these, however, will be found always to destroy one another when substituted in the general term.

Let us next suppose the fraction which produces **a** re­curring series to be

*a* 4. *bx* 4- ear 1 — *ax — βx2 — γxs'*

Let x = -, x = -, x = - be the three roots of the cubic *P I r*

equation 1 — *ax — βxl — γx3 =* 0, thcn the denominator of the fraction will be the product of the three factors

1 — px, 1 — *qx,* 1 — rx.

We must now assume the fraction equal to the expres­sion

1 — *px* 1 — *qx* 1 — *rx ’*

in which P, Q, R denote quantities which are independent of X.

The three terms of this expression are next to be re­duced to a common denominator and collected into one, and the co-efficients of the powers of x in the numerator of the result are to be put equal to the like powers of *x* in the proposed fraction ; we βhall then have

P + Q + R = β,

*(q* + r) P + ( p-∣- r) Q + (p + *q)* R = — *b, qrP* + prQ + pyR = *c,*

and by these equations the values of P, Q, R may be found.

r . P Q R 1 1 1 .

∙<~∙et η , , -=—-— be now resolved into se-

1 — px 1 — *qx* 1 — rx

ries by division ∙ then, adding the like powers of x in each, \*e have

(P + Q + R) + (Pp + *Q.q* + Rr) *x* + (Pp2 4- Q72 + Rr2) x2 -J-, &c.

for the series which is the development of the fraction *a + bx* + cx,

1 — αx — ∕3xs — yx, and here the general term is evidently

(Pp"-1 + ⅝"-, + Rr"-,) *s^~1 ;* and in thς very same manner may the general term be found in every case in which the denominator of the frac­tion admits of being resolved into unequal factors.

(21.) Let us now suppose the fraction to have the form

~~( Γ + √~~, t^le l^enom\*natθr being the product of two equal

factors ; this fraction cannot be decomposed into other frac­tions the denominators of which are the simple factors of its denominator. We may, however, transform it into two, which shall have their numerators constant quantities by proceeding as follows : Assume the numerator α + *bx* = P + Q(l —px), then, that x may remain indetermi­nate, we must have P + 0 = *a, — pQ. = b,* therefore

Q = --,P = α+-.

*P P*

The assumption of α + 6x=P + Q(l —px) gives us therefore

*a -)- bx \_* P Q

(1 — *pxfi — (* 1 — *px)s* 1 — *px'*

*Now,* putting the first term of the latter side of this equa­tion under the form P (1 —*px)~1,* it is resolved by the binomial theorem into the series

P ( 1 + 2px + 3p5xt + 4ρ3x3 +, &c.) ;

the other fraction, τ———, being expanded into a series, is 1 — px

Q + Qpx + Qp'x2 +, &c.

Therefore the complete development of [p—\*s

P + Q + (2P + Q)px + (3P + Q)p5x2 +, &c. and here the general term is manifestly («P 4- Q)p"-1 x"-1, or, substituting for P and Q their values,

{npα + *(n—* 1) *bjpπ~iχ"-1.*

(22.) In general, whatever be the form of the fraction

from which a recurring scries is derived, to determine the general term we must decompose the fraction into others which may be as simple as possible ; and provided it be ra­tional, and the highest power of x in the numerator at least one degree less than the highest power in the denominator, it may be always decomposed into others having one or other of these two forms

P 5\_\_

1 — *pχ,* (l — *qχ)n,*

in which expressions P, Q, p, and *q,* denote quantities in­dependent of x. Each partial fraction gives a recurring series, the general term of which will be sufficiently obvi­ous ; and as the series belonging to the original fraction is the sum of these series, so also its general term will be the sum of all their general terms.

We have now treated of some of the more general me­thods of summing series which admit of being explained by the common principles of algebra ; but the subject is of great extent, and to treat of it so as to give a tolerable no­tion of its various branches, would require more room than could with propriety be spared in such a work as ours.

(23.) The method of fluxions or differential calculus af­fords a method, almost the only general one we possess, of summing series. The general principles upon which it is applied may be stated briefly as follows. Since the in­tegral of any differential containing one variable quantity may always be expressed by a series, on the contrary, every