the place of a term between C"D", CwD'", any two others, the term itself will, if not exactly, at least be nearly, ex­pressed by PQ, the ordinate to the curve.

As an infinite variety of curves may be found that shall pass through the same given points, in this respect the problem is unlimited; it is, however, convenient to assume such as are simple and tractable. The parabolic class pos­sess these properties, and accordingly they are commonly employed.

Let us then express the ordinates CD, C,D', C''D", C",D"', &c. which are the given terms of the series, by

*∕, t, t", f,* &c.

and the abscissæ AD, AD', AD", AD"', or the numbers de­noting the order of the terms, by

a, a,, a”, a",, &c.

Put *y* for PQ, a term to be interpolated, and *x* for AP its place. Then, considering *x* and *g* as indefinite co-ordi­nates, a parabolic curve that shall pass through the points C, C', C", *Cm, Sec.,* will have for its equation

*y* = A + Bx + Ca∙t + Dr3 +, &c. the number of terms on the right-hand side being supposed equal to that of the given points, and A, B, C, &c. being put to denote constant quantities. To determine these, we must consider that when *x — n,* then *y = t,* and that when *x = n',* then *y — t,* and so on ; therefore, substituting the successive corresponding values of *x* and *y,* we get

*t* = A + Bn + Cn\* + Dn’ +, &c.

*f* = A + Bn' + C√i + Da,3 +, &c.

*t’ =* A + Ba" + *Cn"i* + Da” +, &c.

*r"=* A + Ba” + Ca'"2 + Da”3 +, &c.

*Sec. &c.*

This scries of equations must be continued until their num­ber be the same as that of the co-eflicients A, B, C, D, &c. If we now consider *r, t', f,, See.* and n, *n', n", Sec.* as known, and A, B, C, *Sec.* as unknown quantities, we may determine these last by eliminating them one after another from the above equations, as is taught in Algebra. And the values of A, B, C, &c. being thus determined and substituted in the general equation, we shall have a general expression for *y* in terms of *x,* the number denoting its place and known quantities; and this is in substance the solution originally given of the problem by Sir Isaac Newton, who proposed it in the third book of his *Principia,* with a view to its appli­cation in astronomy.

A celebrated foreign mathematician (Lagrange) has, in the *Cahiers de P Ecole Normale,* given a different form to the expression for *y.* He has observed that since, when *x* becomes *n, n', n'', n"', Sec.* successively, then *y* becomes *t, ∕', t", t'", See.* It follows that the expression for *y* must have this form,

*y = at + βf + γtn + H'" +, Sec.* where the quantities *a, β, γ, Sec.* must be such functions of *x,* that if we put *x* = », then *a* = 1 and *β* = 0, χ = 0, *Sec.* and if we put *x = n',* then *a* = 0, *3* = 1, 7 = 0, &c. ; and again, if we make *x = n",* then *x* = 0, *β* = 0, *γ —* 1, *Sec.* and so on. Hence it is easy to conclude that the values of *a, β, γ, Sec.* must have the form

\_ (⅞- n')(τ- w\*)(¾- n")

(a — a’) (a \_ »") (a — a'")’

S - (.«-»)(\*-»')(\* —\*r)

**, (a'-n)(√-n")(π'-n"ζ),**

„ - (\* — »)(» —»9 (¾-\*",) ,ι.

7 ~ (a"-a)(a"-n')a"-a'\*)’ a\_ *(\* —∙) (r-n,) (x — n"') e~*

*~ (n">-n)* (nw — η'7(η’' \_ λc,

Ac.

and here the number of factors in the numerator and deno- minator must be each cqual to the number of given points in the curve. This formula would be found to be identical with that which may be obtained by the method indicated in last article, if we were to take the actual product of the factors and arrange the whole expression according to powers of *x.* It possesses however one advantage over the other, viz. that of admitting of the application of logarithms.

We shall now show the application of this formula.

*Ex.* 1. Having given the logarithms of 101, 102, 104, and 105, it is required to find the logarithm of 103.

In this case we may reckon the terms of the series for­ward from the first given term, viz. log. 101, so that we have

*t* = log. 101 = 2∙0043214, a = 0,

*f =* log. 102 = 20086002, »' = 1,

ry = log. 103 = term sought, *x — 2,*

*t’* = log. 104 = 2∙0170333, *n∙ —* 3,

*f =* log. 105 = 20211893, *n” =* 4.

Substituting now in the general formula, we get

l×-1×-2 \_ I 2×l×-2\_ 2

1×-S×-4- 6’ 7-8×2x- 1- 3’ g\_ 2χ-I X —2 \_ 2 2χlχ-l I

p-l×-2χ-3 - S’ 4×S×1 - 6’

-n. r e . 2f . 2l' e'

Thereforey = -- + -+---

= f(∕ + ∕')-⅛(< + <'")

= 2∙0128372, the answer.

*Ex.* 2. Given a comet’s distance from the sun on the fol­lowing days in December, at twelve at night, to find its distance December 20.

December 12. distance 301, Dec. 24. distance 715,

21. 620, 26. 772.

Here we shall estimate the places of the terms from the time of the first position, viz. December 12. Therefore

*r* = 301, a = 0,

*y* is sought, *x* = 8,

*t' =* 620, ri — 9,

<" = 715, a" =12,

*tn,* = 772, rΓ = 14.

In this case the general formula gives us

\_ 1 «\_ 6 \_ 2 . 8

“ ~ 63’ β ~ 45’ 7 - 3, a - 35’

therefore

y ~ 63 ÷ 45 3 + 35 '

= 586∙3, the answer.

We shall conclude this article with a brief enumeration of the best works on the subject which we have been treat­ing of.

. Bernoulli, *Ars Conjectandi ;* Newton, *Methodus Diffe- rentialis ;* Taylor, *Methodus Incrementorum ;* Stirling, *Me­thodus Differentiatis, sive Tractatus de Summations et In­terpolatione Serierurm ;* Euler, *Institutiones calculi Differen­tiatis ;* Emerson’s Method of Increments ; Emerson’s Dif­ferential Method ; De Moivre, *Miscellanea Analytica* ; La­grange, *Theorie des Fonctions Analytiques ; Traite des Dif­férences et des Séries,* a sequel to Lacroix’s work on the C*al­cul Différentiel;* Dr Hutton’s Mathematical and Philoso­phical Tracts ; Spence’s Essay on the Theory of the various orders of Logarithmic Transcendents, with an Inquiry into their Applications to the Integral Calculus, and the Sum­mation of Series.