after long meditation on the subject that he lost all hopes of success, and contented himself with that approximation to the ratio of the diameter to the circumference which is contained in his treatise *De Circuli Dimensione.* He found his approximation to the ratio, by supposing a regular polygon of ninety-six sides, to be described about the circle, and another of the same number, to be inscribed in it, and by shewing that the perimeter of the circumscribing polygon was less than 310/70, or 31/7 times the diameter, but that the perimeter of the inscribed figure was greater than 310/71 times the diameter ; now, the circumference of the circle being less than the perimeter of the one polygon, but greater than that of the other ; it follows that the circumference must be less than 31/7 times the diameter, but greater than 310/71 times ; so that, taking the first of these limits, as being expressed by the smallest numbers, the circumference will be to the diameter as 31/7 to 1, or as 22 to 7 nearly.

Although the ratio found by Archimedes be the oldest known in the western world, yet one more accurate was known at a much earlier period in India. This we learn from the Institutes of Akbar *(Ayeen Akberry),* where it is said that the Hindoos suppose the diameter of a circle to be to its circumference as 1250 to 3927. Now, this ratio must have required the inscription of a polygon of 768 sides in the circle, and must have been attended with nine ex tractions of the square root, each carried as far as ten places of figures.

We learn from Simplicius that Nicomedes and Apollonius both attempted to square the circle, the former by means of a curve, which he called the *quadratrix ;* the invention of which, however, is ascribed to Dinostratus, and the latter, by the help of a curve, denominated the sister to the tortuous line, or *spiral,* and which was probably the quadratrix of Dinostratus ; the nature of which, and the manner of its application to the subject in question, we shall briefly explain.

Let AFB be a quadrant of a circle,

(fig. 2), and C its centre; and conceive the radius CF to revolve uniformly about C, from the position CA, until at last it coincide with CB ; while at the same time a line DG is carried with an uni­form motion from A towards CB ; the former line continuing always parallel to the latter, until at last they coincide; both motions being supposed to begin and end at the same instant ; the point E, in which the revolving radius CF, and the moveable line DG intersect one another, will generate a certain curve line AEH, which is the *quadratrix* of Dinostratus.

Draw EK, FL both perpendicular to CB ; then because the radius AC and the quadrantal arch AFB, are uniformly generated in the same time by the points D and F, the contemporaneous spaces described will have to one another the same ratio as the whole spaces ; that is, AD : AF : : AC : AB ; hence we have AC : AB : : DC, or EK : FB. Now, as the moveable point F approaches to B, the ratio of the straight line EK to the arch FB will approach to, and will manifestly be ultimately the same as the ratio of the straight line EK to the straight line FL, which again is equal to the ratio of CE to CF ; therefore the ratio of the radius AC to the quadrantal arch AFB is the limit of the ratio of CEtoCF,and consequently equal to the ratio of CH to CB, H being the point in which the quadratrix meets CB. Since therefore CH : CB : : CA, or CB : quad, arch AFB, if by any means we could determine the point H, we might then find a straight line equal to the quadrantal arch, (by finding a third proportional to CH and CB), and consequently a straight line equal to the circumference. The point H, however, can not be determined by a geometrical construction, and there­

fore all the ingenuity evinced by the person who first thought of this method of rectifying the circle, (which certainly is considerable), has been unavailing.

The Arabs, who succeeded the Greeks in the cultivation of the sciences, would no doubt have their pretended squavers of the circle. We however know nothing more than that one of them believed he had discovered that the diameter being unity, the circumfernnce was the square root of 10; a very gross mistake; for the square root of 10 exceeds 3162 ; but Archimedes had demonstrated that the circumference was less than 3·143.

It appears that, during the dark ages, some attempts were made at the resolution of this famous problem, which, how ever, have always remained in manuscripts, buried in the dust of old libraries. But upon the revival of learning, the problem was again agitated by different writers, and particularly by the celebrated Cardinal De Cusa, who distinguished himself by his unfortunate attempt to resolve it. His mode of investigation, which had no solid foundation in geometry, led him to conclude, that if a line equal to the sum of the radius of a circle, and the side of its inscribed square were made the diameter of another circle, and an equilateral triangle were inscribed in this last, the perime ter of this triangle would be equal to the circumference of the other circle. This pretended quadrature was refuted by Regiomontanus.

It would be trespassing too much upon the patience of our readers, were we to mention all the absurd and erroneous attempts which have been made during the last three centuries to square the circle. In a supplement to Montucla’s *Histoire des Mathematiques,* we find upwards of forty pretenders to the honour of this discovery enumerated. They were almost all very ignorant of geometry; and many of them were wild visionaries, pretending to discover inexplicable relations between the plain truths of mathematics and the most mysterious doctrines of religion. From such persons as have generally pursued this inquiry, no improve­ment whatever of the science was to be expected ; al­though, in some instances, it has derived advantage from the labours of euch as have undertaken to expose the ab surdity of their conclusions ; as in the case of Metius, who in refuting the quadrature of one Simon à Quercu, found a much nearer approximation to the ratio of the diameter to the circumference than had been previously known in Eu rope, namely, that of 113 to 355.

Among the most remarkable of those who have under taken to resolve this problem, we cannot fail to enumerate Joseph Scaliger, a man of stupendous erudition. Full of confidence in his own powers, he believed that, entering upon the study of geometry, he could not fail to surmount by the force of his genius, those obstacles which had completely stopped the progress of all preceding inquirers. He gave the result of his meditations to the world in 1592, under the title *Nova Cyclometria ;* but he was refuted by Clavius, by Vieta, and others, who shewed that the magnitude which he had assigned to the circumference was a little *less* than the peri meter of the inscribed polygon of 192 sides, which proved beyond a doubt that he was wrong. Scaliger, however, was not to be convinced of the absurdity of his conclusion ; and indeed, in almost every instance, pretenders to this dis covery have not been more remarkable for their egregious errors, than for their obstinacy in maintaining that they were in the right, and all who held a contrary opinion in the wrong.

The famous Hobbes came also upon the field about the year 1650, with pretensions not only to the quadrature of the circle, but also to the trifection of an angle, the reclification of the parabola, &c. ; but his pretended solutions were refuted by Dr. Wallis. And this circumstance aftbrd ed him occasion to write not only against geometers, but even against the science of geometry itself.