We find it recorded by Montucla, as a sort of phenome­non, that one Richard White, an English Jesuit, having happened upon what he conceived to be a quadrature of the circle, which he published under the title, *Chrysæspis, seu Quadratura Circuli,* suffered himself at last to be convinced by some of his friends that he was wrong. But a solution of the same problem, found out by one Mathulen of Lyon, did not end in so much advantage to its author. This man in 1728 announced to the world that he had discovered both the quadrature of the circle, and a perpetual motion ; and he was so certain of the truth of these discoveries, that he he consigned 1000 *ecus* (about L.125), to be paid to any one who should demonstrate that he was deceived in either. The task was not difficult. Nicole of the Academy of Sciences demonstrated that he was wrong, and he himself allowed it ; but he hesitated to pay the money, which Nicole had relinquished in favour of the Hotel Dieu of Lyon. The affιir went before a court of justice, which adjudged the money to be paid, as Nicole had destined it, to the poor. At a later period, namely, in 1753, the Chevalier de Causons, a French officer, and a man who was never sup posed to be a mathematician, suddenly found a quadrature of the circle in procuring a circular piece of turf to be cut ; and rising from one truth to another, he explained by his quadrature the doctrine of original sin, and the Trinity. He engaged himself, by a public writing, to deposit with a notary the sum of 300,000 francs, to be wagered against such as should oppose him, and he actually lodged 10,000, which were to devolve to him who should demonstrate his error. This was easily done, as it resulted from his disco very that a circle was equal to its circumscribing square, that is, a part to the whole. Some persons came forward to answer his challenge, and in particular a young lady sued him at one of the courts of law; but the French king judg ed that the chevalier’s fortune ought not to suffer on ac count of his whim ; for, setting aside this piece of folly, in every other respect he was a worthy man. The procedure was therefore stopped, and the wager declared void.

We shall not enter further into the history of these vain and absurd attempts to resolve this important problem, but proceed to state what has actually been done by men of sound minds and real mathematical acquirements towards its solution. And in the first place, it may be observed that the problem admits of being proposed under two diffc rent forms ; for it may be required to find either the area of the whole circle, or, which is the same thing, the length of the whole circumference ; or else to find the area of any proposed sector or segment, or, which is equivalent, the length of the arch of the sector or segment. The former is termed the *definite,* and the latter the *indefinite* quadrature of the circle. The latter evidently is more general than the former, and includes it as a particular case. Now, if we could find by any means a finite algebraic equation that should express the relation between any proposed arch of a circle, and some known straight line, or lines, the magni­tude of one or more of which depended on that arch, then we would have an absolute rectification of the arch, and consequently a rectification or quadrature of the whole cir­cle. We here speak of an analytical solution ; the ancients, who were almost entirely ignorant of this branch of mathematical science, must have endeavoured to treat it entirely upon geometrical principles. It is now well known, how ever, that all geometrical problems may be subjected to analysis ; and that it is only by such a mode of proceeding they have in many cases been resolved.

With respect to the definite quadrature of the circle, no unexceptionable demonstration of its impossibility has hi­therto been published. It is true, that James Gregory, in his *Vera Circuli et Hyperbola Quadratura,* has given what he considered as such a demonstration ; but it was objected to by Huygens, one of the best geometers of his time. It

is certain that the ratio of the diameter to the circumference, also, that the ratio of the square of the diameter to the square of a straight line equal to the circumference, cannot be expressed by rational numbers, for this has been strictly demonstrated by Lambert in the Berlin Memoirs for 1761. A demonstration is also given in Legendre’s *Geo­metrie.* As to the indefinite quadrature, if Newton’s demonstration of the twenty-eighth lemma of the first book of his *Principia,* be correct, the thing ought to be absolutely impossible. For the object of that proposition is to prove, that in no oval figure whatever, that returns into itself, can the area cut off by straight lines at pleasure, be universally found by an equation of a finite dimension, and composed of a finite number of terms. If this be true, then it will be impossible to express any sector of a circle taken at plea­sure in finite terms. It is however to be remarked, that the accuracy of the reasoning by which Newton has attempted to establish the truth of the general proposition, has been questioned by no less a geometer than D’Alembert ; and in deed we know one oval curve, which returns into itself, and which according to Newton’s proposition ought there fore not to admit of an indefinite quadrature ; yet this is by no means the case, for it does really admit of such a quad rature. The curve we mean is the *lemniscata,* whose figure is nearly that of the numeral character 8. Upon the whole, then, we may infer that an unexceptionable demonstration of the impossibility of expressing either the whole circle, or any proposed sector of it, by a finite equation, is still among the *desiderata* of mathematics.

Of the different methods which have been found for approximating to the area or to the circumference, that of Archimedes is the only one found by the ancients in the western world that has descended to modern times, and it appears to have been the most accurate known, until about the year 1585, when Metius, in refuting a pretended quad­rature, found the more accurate ratio of 113 to 355, as we have already noticed. About the same time Vieta and Adrianus Romanus published their ratios, the former carrying the approximation to ten decimals instead of six, (which was that of Metius’s ratio), and the latter extending it to 17 figures. Vieta also gave a kind of series, which being continued to infinity, gave the value of the circle.

These approximations, however, were far exceeded by that of Ludolph Van Ceulen, who, in a work published in Dutch in 1610, carried it as far as 36 figures, showing that if the diameter were unity, the circumference would be greater than 3.14159,26535,89793,23846,26433,83279,50288, but less than the same number with the last figure increased by an unit. In finding this approximation, Van Ceulen fol­lowed the method of Archimedes, doubling continually the number of sides of the inscribed and circumscribed poly gons, until at length he found two which differed only by an unit in the 36th place of decimals in the numbers ex pressing their perimeters. This was rather a work of patience than of genius ; indeed the labour must have been prodigious. He seems to have valued highly this singular effort; for in imitation of Archimedes, whose tomb was adorned with a sphere and cylinder, he directed that the ratio he had found might be inscribed on his tomb.

Snellius abridged greatly the labour of calculation ; and although he did not go beyond Van Ceulen, yet he verified his result. Des Cartes also found a geometrical construc­tion, which being repeated continually, gave the circum­ference, and from which he might easily have deduced an expression in the form of a series. Gregory of St. Vincent distinguished himself also on this subject ; he however committed a great error in supposing he had dis covered the quadrature of both the circle and hyperbola. Gregory’s mistake was the cause of a sharp controversy carried on between his disciples on the one side, and by Huygens, Mersenne, and Lestaud, on the other ; and it