was this that gave Huygens occasion to consider particularly the quadrature of the circle, and to investigate various new and curious theorems relating to it, which are contained in his *Theoremata de Quadratura Hyperboles, El­lipsis, et Circuli,* 1651 ; and in his work *De Circuli Magni­tudine inventa,* 1654. In particular he showed, that if c denote the chord of an arch, and *s* its sine, then the arch itself will be greater than c+1/3(c—s), but less titan c+ *4c* + *s*

——— ×½(c—i): he also showed that the arch is less than 2e+3jt a

the sum of 2/3 of its sine and 1/3 of its tangent.

James Gregory, in his *Vera Circuli et Hyperbolæ Quad­*

*ratura,* gave several curious theorems upon the relation of the circle to its inscribed and circumscribed polygons, and their ratios to one another ; and by means of these he found with infinitely less trouble than by the ordinary methods, and even by those of Snellius, the measure of the circle as far as 20 places of figures. He gave also, after the example of Huygens, constructions for finding straight lines nearly equal to arches of a circle, and of which the degree of accuracy was greater. For example, he found that if A be put for the chord of an arch of a circle, and B for twice the chord of half the arch, and C be taken such that A+B : B : : B : C, then the arch itself is nearly equal to

—∆j but a little less, the error in the case of a 15

complete semicircle being less than its jj>δβ part; and when the arch does not exceed 120°, it is less than its 1/400000 part ; and finally, for a quadrant the error is not greater than its 1/300000 part. And farther, that if D be such that A : B : :

„ v v . , , 12C + 4B—D ,

B : D, then the arch is nearly equal to ⅛, , but

J 5

a little greater, the error in the semicircle being less than its 1/1000 part, and in a quadrant less than its 1/60000 part.

Dr. Wallis gave, in his *Arithmetica Infinitorum,* a singular expression for the ratio of the circle to the square of its diameter. He found that the former was to the latter as 1 to the product

3×3×5×5×7×7×9×9× 11 ×I1, &c. 2×4×4×6×6×8×8×10×10×12

the fractions 5/2, 5/4, 5/4, 5/6, &c., being supposed infinite in num­ber. The products being supposed continued to infinity, we have the ratio exactly ; but if we stop at any finite num­ber of terms, as must necessarily be the case in its application, the result will be alternately too great and too small, according as we take an odd or an even number of terms of the numerator and denominator.

An expression of another kind for the ratio of the circle to the square of the diameter was found by Lord Brounker. He showed that the circle being unity, the square of the diameter is expressed by the continued fraction 1

H 9

*2+*

*2+∞*

2+l?\_

2+,&c

which is supposed to go on to infinity, the numerators 1, 9, 25, 49, *Sic.,* being the squares of the odd numbers 1, 3, 5, 7, &c. By taking two, three, four, &c., terms of this fraction, we shall have a scries of approximate values, which are alternately greater and less than its accurate value.

Such were the chief discoveries relating to the quadrature of the circle made before the time of Newton ; many others, however, were quickly added by that truly great

man, as well as by his contemporaries. In particular, James Gregory found, that *t* being put for the tangent, the arch is expressed by the very simple series

*t3 ti f . ti*

t—3^^^ 5—7" + 9— &c

By supposing that *t*=1, in which case the arch is one eighth of the circumference, we have the corresponding arch expressed by the series

4<⅛M+⅛+\*c∙>

which was also given by Leibnitz as a quadrature of the circle in the *Acta Eruditorum* in the year 1682, but was dis covered by him 1673. Gregory had however found the series under its general form several years before. This series is altogether inapplicable in its present form, on ac count of the slowness of its convergency ; for Newton has observed, that to exhibit its value exact to twenty places of figures, there would be occasion for no less than five thou sand millions of its terms, to compute which would take up above a thousand years.

The slowness of the convergency has arisen from our supposing *t*=1. If we had supposed *t* greater than 1, then the series would not have converged at all, but on the contrary diverged. But by giving to *t* a value less than 1, then the rate of convergency will be increased, and that so much the more, as *t* is smaller.

If we suppose the arch of which *t* is the tangent to be 30°, then *t* will be √1/3=1/3√3, and therefore half the circumference to radius unity, or the circumference to the diameter unity, will be

\*∕12(1X3 +533~73s + 9∑∙&C')·

Mr. Machin, enticed by the easiness of the process, was induced, about the beginning of the last century, to continue the approximation as far as 100 places of figures, thus finding the diameter to be to the circumference as 1 to 3.14159,26535,89793,23846,26433,83279,50288,41971,69 399,37510,58209,74944,59230,78164,06286,20899,86280, 34825,34211,70680. After him, De Lagny continued it as far as 128 figures. But he has also been outdone; for in Radcliffe’s library at Oxford, there is a manuscript in which it is carried as far as 150 figures !

Although this last series, which was first proposed by Dr. Halley, gives the ratio of the diameter to the circumference with wonderful facility when compared with the operose method employed by Van Ceulen, yet others have been since found which accomplish it with still greater ease.

We have given such a method in our treatise on Algebra, (see article 272.) In the same treatise, (article 273), we have given a very elementary method, which may be understood by the mere elements of geometry. For the analytical methods, see Fluxions.

SRAVANA BALGUTA, a village of Hindustan, in the northern province of Mysore, celebrated as the principal seat of the Jain worship, which at one time prevailed all over the south of India, but which has now greatly declined, from the influence of the Brahmins. There arc two hills adjoining the town, on one of which, named Indra Betta, is situated the temple ; the other is cut into a colossal statue, seventy feet in height. The village is almost wholly inhabited by Jainas. Long. 76. 43. E. Lat. 12. 45. N.

SRI MUTTRA, a considerable town of Hindustan, pro vince of Agra. It is situated on a naked rock of red gra nite, of which material all the houses are constructed. The streets are very narrow, and the working of the red stone into slabs furnishes employment to the greater part of the inhabitants. Long. 77. 20. E. Lat. 26. 41. N.