■ pose that the crooked lever virtually concerned in the strain is DAB. We must find the point I, which is the centre of effort of all the attractive forces, or that point where the full cohesion of AD must be applied, so as to have a mo­mentum equal to the accumulated momenta of all the variable forces. We must in like manner find the centre of effort *i* of the repulsive or supporting forces exerted by the fibres lying between A and Δ.

It is plain, and the remark is important, that this last centre of effort is the real fulcrum of the lever, although A is the point where there is neither extension nor contrac­tion ; for the lever is supported in the same manner as if the repulsions of the whole line A∆ were exerted at that point. Therefore let S represent the surface of fracture from A to D, and *f* represent the absolute cohesion of a fibre at D in the instant of fracture. We shall have ∕S × I 4- *i = pl,* or *I* : I -∣- *i =fS : p ;* that is, the length AB is to the distance between the two centres of effort I and *i*, as the absolute cohesion of the section between A and D is to the relative strength of the section.

It would be perhaps more accurate to make Al and Ai equal to the distances of A from the horizontal lines passing through the centres of gravity of the triangles of dAD and δA∆. It is only in this construction that the points I and *i are* the centres of real effort of the accumulated attrac­tions and repulsions. But I and i, determined as we have done, are the points where the full equal actions may be all applied, so as to produce the same momenta. The final results are the same in both cases. The attentive and duly informed reader will see that Mr Bullinger, in a very elabo­rate dissertation on the strength of beams, in the *Comment. Petropolitan.* 1729, has committed several mistakes in his estimation of the actions of the fibres. We mention this because his reasonings are quoted and appealed to as au­thorities by Muschenbroeck and other authors of note. The subject has been considered by many authors on the con­tinent. We recommend to the reader’s perusal the very minute discussions in the Memoirs of the Academy of Paris for 1702 by Varignon, the Memoirs for 1708 by Pa­rent, and particularly that of Coulomb in the *Mém. par les Sçavans Etrangers,* tom. vii.

It is evident from what has been said above, that if S and *s* represent the surfaces of the sections above and below A, and if G and *g* are the distances of their centres of gra­vity from A, and O and *o* the distances of their centres of oscillation, and D and *d* their whole depths, the momentum of cohesion will be—*0- —pl.*

1J *d t*

If, as is most likely, the forces are proportional to the extensions and compressions, the distances AI and At, (τ \* O

which are respectively = —θ- and 9~, are respectively = 1/3 DA and 1/3 ∆A, and when taken together are = 1/3 D∆. If, moreover, the extensions are equal to the compressions in the instant of fracture, and the body is a rectangular prism like a common joist or beam, then DA and ∆A are also equal ; and therefore the momentum of cohesion is *f b × 1/2 d × 1/3 d = f b d* × 1/6 *d - pl.* Hence we

obtain this analogy : “ six times the length is to the depth as the absolute cohesion of the section is to its relative strength.”

Thus we see that the compressibility of bodies has a very great influence on their power of withstanding a trans­verse strain. We see that in this most favourable suppo­sition of equal dilatations and compressions, the strength is reduced to one half of the value of what it would have been had the body been incompressible. This is by no means obvious ; for it does not readily appear how compressibi­

lity, which does not diminish the cohesion of a single fibre, should impair the strength of the whole. The reason, how­ever, is sufficiently convincing when pointed out. In the instant of fracture, a smaller portion of the section is actually exerting cohesive forces, while a part of it is only serving as a fulcrum to the lever by whose means the strain on the section is produced. We see, too, that this diminution of strength does not so much depend on the sensible com­pressibility, as on its proportion to the dilatability by equal forces. When this proportion is small, A∆ is small in com­parison of AD. and a greater portion of the whole fibre is exerting attractive forces. The experiments already men­tioned, of Duhamel de Monceau, on battens of willow, show that its compressibility is nearly equal to its dilatabi­lity. But the case is not very different in tempered steel. The famous Harrison, in the delicate experiments which he made while occupied in making his longitude watch, discovered that a rod of tempered steel was nearly as much diminished in its length as it was augmented by the same external force. But it is not by any means certain that this is the proportion of dilatation and compression which obtains in the very instant of fracture. We rather imagine that it is not. The forces arc nearly as the dilatations till very near breaking; but we think that they diminish when the body is just going to break. But it seems certain that the forces which resist compression increase faster than the compressions, even before fracture. We know incontes­tably that the ultimate resistances to compression are in­superable by any force which we can employ. The repul­sive forces, therefore, in their whole extent, increase faster than the compressions, and are expressed by an assymp- totic branch of the Boscovician curve formerly explained. It is therefore probable, especially in the more simple sub­stances, that they increase faster, even in such compres­sions as frequently obtain in the breaking of hard bodies. We ate disposed to think that this is always the case in such bodies as do not fly off in splinters on the concave side; but this must be understood with the exception of the permanent changes which may be made by compres­sion when the bodies are crippled by it. This always in­creases the compression itself, and causes the neutral point to shift still more towards D. The effect of this is some­times very great and fatal.

Experiment alone can help us to discover the proportion between the dilatability and compressibility of bodies. The strain now under consideration seems the best calculated for this research. Thus if we find that a piece of wood an inch square requires 12,000 pounds to tear it asunder by a direct pull, and that 200 pounds will break it transversely by acting 10 inches from the section of fracture, we must conclude that the neutral point A is in the middle of the depth, and that the attractive and repulsive forces are equal. Any notions that we can form of the constitution of such fibrous bodies as timber, make us imagine that the *sensible* compressions, including what arises from the bending up of the compressed fibres, is much greater than the real cor­puscular extensions. One may get a general conviction of this unexpected proposition by reflecting on what must happen during the fracture. An undulated fibre can only be drawn straight, and then the corpuscular extension be­gins ; but it may be bent up by compression to any degree, the corpuscular compression being little affected all the while. This observation is very important; and though the forces of corpuscular repulsion may be almost insupe­rable by any compression that we can employ, a *sensible* compression may be produced by forces not enormous, suf­ficient to cripple the beam. Of this we shall see very im­portant instances afterwards.

It deserves to be noticed,, that although the relative strength of a prismatic solid is extremely different in the three hypotheses now considered, yet the proportional