strengths of different pieces follow the same ratio, namely, the direct ratio of the breadth, the direct ratio of the square of the depth, and the inverse ratio of the length. In the first hypothesis (of equal forces) the strength of a rectangu­lar beam was -^^- ; in the second (of attractive forces pro­portioned to the extensions) it was '^gγ' ; and in the third (equal attractions and repulsions proportional to the exten­sions and compressions) it was -η^-, or more generally ^-γ, where *m* expresses the unknown proportion between the at­tractions and repulsions corresponding to an equal exten­sion and compression.

Hence we derive a piece of useful information, which is confirmed by unexceptionable experience, that the strength of a piece depends chiefly on its depth, that is, on that di­mension which is in the direction of the strain. A bar of timber of one inch in breadth and two inches in depth is four times as strong as a bar only one inch deep, and it is twice as strong as a bar two inches broad and one deep ; that is, a joist or lever is always strongest when laid on its edge.

There is therefore a choice in the manner in which the cohesion is opposed to the strain. The general aim must be to put the centre of effort I as far from the fulcrum or the neutral point A as possible, so as to give the greatest energy or momentum to the cohesion. Thus if a triangular bar projecting from a wall is loaded with a weight at its ex­tremity, it will bear thrice as much when one of the sides is uppermost as when it is undermost.

Hence it follows that the strongest joist that can be cut out of a round tree is not the one which has the greatest quantity of timber in it, but such that the product of its breadth by the square of its depth shall be the greatest pos­sible. Let ABCD (fig. 7) be the sec­tion of this joist inscribed in the circle, A B being the breadth and AD the depth. Since it is a rectangular section, the di­agonal BD is a diameter of the circle, and BAD is a right-angled triangle. Let BD be called *a,* and BA be called *X ;* then A D = √ *a't—x3.* Now we must have AB ∙ AD2, or *x (a2 —* x2), or *a2- x — x3,* a minimum ; its differential (*a2* — 3x2) *dx* must be 0, or *a2 —* 3x2, or *x2 —* If therefore we make DE = 1/3

DB, and draw EC perpendicular to BD, it will cut the cir­cumference in the point C, which determines the depth BC and the breadth CD.

Because BD : BC = CD : CE, we have the area of the section BC·CD = BD·CE. Therefore the different sections having the same diagonal BD, are proportional to their heights CE. Therefore the section BCDA is less than the section B*c* D*a*, whose four sides are equal. The joist so shaped, therefore, is stronger, lighter, and cheaper.

The strength of ABCD is to that of a B*c* D as 10,000 to 9186, and the weight and expense as 10,000 to 10,607 ; so that ABCD is preferable to *a* B *c* D, in the proportion of 10,607 to 9186, or nearly 115 to 100.

From the same principles it follows that a hollow tube is stronger tban a solid rod containing the same quantity of matter. Let fig. 8 re­present the section of a cylindric tube, of which AF and BE are the exterior and interior diameters, and C the centre. Draw BD perpendicular to BC, and join

DC. Then, because BD2 = CD2 —CB2, BD is the radius of a circle containing the same quantity of matter with the ring. If we estimate the strength by the first hypothesis, it is evident that the strength of the tube will be to that of the solid cylinder, whose radius is BD, as BD2 × AC to BD2 × BD ; that is, as AC to BD ; for BD2 expresses the cohesion of the ring of the circle, and AC and BD are equal to distances of the centres of effort (the same with the centres of gravity) of the ring and circle from the axis of the fracture.

The proportion of these strengths will be different in the other hypothesis, and is not easily expressed by a general formula ; but in both it is still more in favour of the ring or hollow tube.

The following very simple solution will be readily under­stood by the intelligent reader. Let O be the centre of oscillation of the exterior circle, *o* the centre of oscillation of the inner circle, and *w* the centre of oscillation of the ring included between them. Let M be the quantity of surface of the exterior circle, *m* that of the inner circle, and *μ* that of the ring. . . , ∣

, r, M∙FO-m∙Fa 5 FCs -4- EC!

W e have 1 *u> =*  . , and

*μ —* 4 FC

∕\* u X Ft0 the strength of the ring = , and tire strength of

the same quantity of matter in the form of a solid cylinder is *f* *μ ×* 5/8 BD ; so that the strength of the ring is to that of the solid rod of equal weight as F *w* to 5/4 BD, or nearly as FC to BD. This will easily appear by recollecting that FO is ~~\_\_ ⅜um~~ ~~ll~~~~\*f~~ ~~r~~ z (see Rotation) and that the momentum of m∙rC

, . . ∕λ∙FC∙Co *f∕n∙Fo c , . . ,*

cohesion is -q∙ ~~p(j~~'^- ~o— for the inner circle, &c.

Emerson has given a very inaccurate approximation to this value in his Mechanics, 4to.

This property of hollow tubes is accompanied also with greater stiffness ; and the superiority in strength and stiff­ness is so much the greater as the surrounding shell is thin­ner in proportion to its diameter.

Here we see the admirable wisdom of the Author of na­ture in forming the bones of animal limbs hollow. The bones of the arms and legs have to perform the office of levers, and are thus opposed to very great transverse strains. By this form they become incomparably stronger and stiff­er, and give more room for the insertion of muscles, while they are lighter and therefore more agile ; and the same wisdom has made use of this hollow for other valuable pur­poses of the animal economy. In like manner the quills in the wings of birds acquire by their thinness the very great strength which is necessary, while they are so light as to give sufficient buoyancy to the animal in the rare medium in which it must live and fly about. The stalks of many plants, such as all the grasses, and many reeds, are in like manner hollow, and thus possess an extraordinary strength. Our best engineers now begin to imitate nature by making many parts of their machines hollow, such as their axles of cast iron, &c. ; and the ingenious Mr Ramsden made the axes and framings of his great astronomical instruments in the same manner.

In the supposition of homogeneous texture, it is plain that the fracture happens as soon as the particles at D are separated beyond their utmost limit of cohesion. This is a determined quantity, and the piece bends till this de­gree of extension is produced in the uttermost fibre. It follows, that the smaller we suppose the distance between A and D, the greater will be the curvature which the beam will acquire before it breaks. Greater depth therefore makes a beam not only stronger, but also stiffer. But if the parallel fibres can slide on each other, both the strength and the stiffness will be diminished. Therefore, if, instead of one beam D∆KC (fig. 6), we suppose two, DABC and