arm OL and Oλ. must be equally strong in all its parts. The longitudinal sections of each arm must be a triangle, a common parabola, or a cubic parabola, according to the conditions previously given.

And, moreover, all these forms are equally strong; for any one of them is equally strong in all its parts, and they are all supposed to have the same section at the front of the wall or at the fulcrum. They are not, however, equally stiff. The first, represented in fig. 12, will bend least upon the whole, and the one formed by the cubic parabola will bend most. But their curvature at the very fulcrum will be the same in all.

It is also plain, that if the lever is of the second or third kind, that is, having the fulcrum at one extremity, it must still be of the same shape ; for in abstract mechanics it is indifferent which of the three points is considered as the axis of motion. In every lever the two forces at the ex­tremities act in one direction, and the force in the middle acts in the opposite direction, and the great strain is always at that point. Therefore a lever such as fig. 12, moveable round an axis passing horizontally through λ, and acting against an obstacle at OP, is equally able in all its parts to resist the strains excited in those parts.

The same principles and the same construction will ap­ply to beams, such as joists, supported at the ends L and λ (fig. 12), and loaded at some intermediate part OP. This will appear evident by merely inverting the directions of the forces at these three points, or by recurring to tire ar­ticle Roof, p. 444.

Hitherto we have supposed the external straining force as acting only in one point of the beam. But it may be uniformly distributed all over the beam. To make a beam in such circumstances equally strong in all its parts, the shape must be considerably different from the former.

Thus suppose the beam to project from a wall. If it be of equal breadth throughout, its sides being vertical planes parallel to each other and to the length, the ver­tical section in the direction of its length must be a tri­angle instead of a common parabola ; for the weight uni­formly distributed over the part lying beyond any section, is as the length beyond that section : and since it may all be conceived as collected at its centre of gravity, which is the middle of that length, the lever by which this load acts or strains the section is also proportioned to the same length. The strain on the section (or momentum of the load) is as the square of that length. The section must have strength in the same proportion. Its strength being as the breadth and the square of the depth, and the breadth being con­stant, the square of the depth of any section must be as the square of its distance from the end, and the depth must be as that distance ; and therefore the longitudinal vertical section must be a triangle.

But if all the transverse sections are circles, squares, or any other similar figures, the strength of every section, or the cube of the diameter, must be as the square of the lengths beyond that section, or the square of its distance from the end ; and the sides of the beam must be a semi- cubical parabola.

If the upper and under surfaces are horizontal planes, it is evident that the breadth must be as the square of the dis­tance from the end, and the horizontal sections may be formed by arches of the common parabola, having the length for their tangent at the vertex.

By recurring to the analogy so often quoted between a projecting beam and a joist, we may determine the proper form of joists which are uniformly loaded through their whole length.

This is a frequent and important case, being the office of joists, rafters, &c. ; and there are some circumstances which must be particularly noticed, because they are not so ob­vious, and have been misunderstood. When a beam AB (fig. 14) is supported at the ends, and a weight is laid on any point P, a strain is excited in every part of the beam.

The load on P causes the beam to press on A and B, and the props re-act with forces equal and opposite to these pressures. The load at P is to the pressures at A and B as A B to PB and PA, and the pressure at A is to that at B as BP to PA ; the beam therefore is in the same state, with respect to strain in every part of it, as if it were rest­ing on a prop at P, and were loaded at the ends with weights equal to the two pressures on the props : and ob­serve, these pressures are such as will balance each other, being inversely as their distances from P. Let P repre­sent the weight or load at P. The pressure on the prop P PA

must be P × PA/AB; This is therefore the re-action of the prop

B, and is the weight which we may suppose suspended at B, when we conceive the beam resting on a prop at P, and carrying the balancing weights at A and B.

The strain occasioned at any other point C, by the load P at P, is the same with the strain at C, by the weight P × PA/AB hanging at B, when the beam rests on P, in the manner now supposed ; and it is the same if the beam, in­stead of being balanced on a prop at P, had its part AP fixed in a wall. This is evident. Now we have shown at length that the strain at C, by the weight at B, is P × PA/AB × BC. We desire it to be particularly re­marked, that the pressure at A has no influence on the strain at C, arising from the action of any load between A and C ; for it is indifferent how the part AP of the projecting beam PB is supported. The weight at A just performs the same office with the wall in which we suppose the beam to be fixed. We are thus particular, because we have seen even persons not unaccustomed to discussions of this kind puzzled in their conceptions of this strain.

Now let the load P be laid on some point *p* between C and B. The same reasoning shows us that the point is, with respect to strain, in the same state as if the beam were fixed in a wall, embracing the part *p*B, and a weight = P × pB/AB were hung on at A, and the strain at C is P A b

**× pB/AB** × AC.

In general, therefore, the strain on any point C, arising from a load P laid on another point P, is proportional to the rectangle of the distances of P and C from the ends nearest to each. It is P × PA×CB/AB, or P×*p*B×CA/AB, according as the load lies between C and A or between C and B.

*Cor.* 1. The strains which a load on any point P occa­sions on the points C, c, lying on the same side of P, are as the distances of these points from the end B. In like manner the strains on E and *e* arc as EA and eA.

*Cor.* 2. The strain which a load occasions in the part on which it rests is as the rectangle of the parts on each side. Thus the strain occasioned at C by a load is to that at D by the same load as AC × CB to AD × DB. It is there­fore greatest in the middle.

Let us now consider the strain on any point C arising from a load uniformly distributed along the beam. Let AP be re­presented by *x,* and P*p* by *dx,* and the whole weight on the beam by *a.* Then