In the examples of the mensuration of heights, the verti­cal line to be measured has been supposed to stand on the horizontal plane of the station, where the angle of eleva­tion was taken ; but in estimating the difference between the heights of two stations which are at a considerable dis­tance from each other, this is not exactly true : in such a case it is necessary to correct the observed vertical angle, on account of the earth’s curvature.

Prob. II.

Having given the distance between two stations, and the elevation or depression of the one as seen from the other, to determine the correction to be made in the ver­tical angle on account of the earth’s curvature.

Let A and B be the two stations (fig. 3), and C the earth’s centre. Draw the horizontal line AH in the plane of the triangle ABC, then BAH will be the ap­parent angle of ele­vation or depression of B, according as it is above or below the horizon of A.

Take CD = CA, and join AD ; the line BD is the dif­ference between the heights of the sta­tions A and B, and to determine BD, the vertical angle BAD must be known ; now this angle differs from the apparent elevation BAH by the angle HAD, therefore HAD is the correction of the vertical angle, de­pending on the earth’s curvature.

In the isosceles triangle ACD, the sum of its angles, that is, 2CAD + C, is equal to two right angles (Geom. 11 and 24, 1 ), and therefore CAD + 1/2C = one right angle ; but CAH is a right angle, therefore CAH = CAD + 1/2C, and taking away the common angle CAD, there remains HAD = 1/2C. Hence the correction on account of the earth’s curvature is *half the arc intercepted between the stations.*

Cor. If the station B be above the horizon, as seen from A, the corrected vertical angle will be the sum of its appa­rent elevation and half C, the angle contained by the ver­ticals passing through the stations ; but if B be below the horizon, then it will be the excess of half C above the ap­parent depression.

*Ex.* In the British trigonometrical survey, the distance between the stations on Wisp Hill and Cross Fell was found to be 235,018∙6 feet, which, reckoning 60941/2 feet to a minute, corresponds to 38' 33"∙7 on the earth’s surface. From Wisp Hill, Cross Fell appeared depressed below its horizon 2' 31"; but, by a corresponding observation at the other station, the error arising from refraction was estimated at 2’ 37"∙3. Hence the difference between the heights of the stations is required.

Let A be Wisp Hill, B Cross Fell. The observed de­pression HAB was 2' 31”; but as the line AB was elevated 2' 37"∙3 by refraction, the correct value of HAB was 2' 31" + 2' 37"∙3 = 5' 8'∙3. This subtracted from HAD = 1/2C = 19' 16"∙8, leaves 14' 8"∙5 for the vertical angle BAD. And because in the triangle BAD the angle D is almost a right angle, therefore BD will be found by this proportion ; rad.: tan. BAD(14'8"∙5).: AD(235,018’6):BD= 966∙8 feet, the height of the station on Cross Fell above that on Wisp Hill.

On a survey, it will sometimes happen that the instru­ment cannot he conveniently placed at the very centre of a station, in order to determine the angles subtended by remote objects : it must then be placed at a point near the station, and the angles taken at that point must be correct­ed by calculation, so as to reduce them to the centre. How this may be done is shown in the next problem.

Prob. III.

To reduce an angle taken out of a station to the centre of the station.

Let C be the centre of a permanent station (fig. 4), where the angle ACB, subtended by two remote objects A and B, is to be determined ; let O be a given point at a little distance, where the instrument is placed, and the angle A OB actually measured ; then, having given the distance CO, and the an­gles COA, COB ; also the dis­tances C A, CB, or at least their values nearly, it is required to find the difference between the angles ACB, A OB.

Let I be the intersection of AC and BO; and because the angle AIB is the sum of ACB and CBO, also the sum of AOB and CAO (Geom. 24, 1) ; therefore

ACB + CBO = AOB + CAO, and ACB — AOB = CAO — CBO.

Now, in the triangles COA, COB,

CA : CO : : sin. COA : sin. CAO,  
CB : CO : : sin. COB : sin. CBO.

From these proportions, the angles CAO, CBO may be found, and their difference, which is also the difference of the angles ACB, AOB will be known.

It is useful in practice to have a formula that expresses the difference of the angles BCA, BOA in minutes of a degree. For this purpose, put the angles

ACB = C, AOB = O, BOC *= v;* then COA = O *v*,∙ also put the lines CA = m, CB = *n,* OC *=d ;*

then the two proportions become

*m : d : :* sin. (O + *v)* : sin. CAO ; *n* : *d* : : sin. *v* : sin. CBO ;

hence sin. CAO = *d/m* sin. (O + *v),* sin. CBO = - sin. *v.*

But small angles being almost proportional to their sines, it follows that the number of minutes in the angle CAO will be sin. CAO/sin. 1 nearly ; and in like manner the number of minutes in the angle CBO will be sin. CBO/sin. 1, nearly ; therefore

CAO = dsin. (O + v)/msin. 1', CBO + dsin. v/nsin. 1';

hence, since CAO —CBO = C — O, we have

*r, r∖ — d*sin· (O -∣- ι∙) *d*sin. *v* t> xj —— —“~~~ , \_ . ” , -∣ ,\*

msιn. 1' nein. 1

This is the correction of the angle O expressed in mi­nutes of a degree ; in its application, attention must be paid to the signs of sin. (O + *v)* and sin. *v*, agreeably to the rales laid down in *Arithmetic of Sines,* Algebra (225).

A different and more simple expression for the correc­tion of the angle O may be found as follows:

Let a circle be described about the triangle ABC, meet­ing BO in H, and join CH, AH. Let the angles BCA, BOA, BOC, and the lines CA, CB, CO, be denoted by the same letters as before ; and in addition, put the angle BAC = A.