tance of an infinitely slender pencil of central rays, and the *, . m—*1 /„ *nιkι∖et ...*

other,—*φ- — lh3* —)-, is the aberration arising

from the spherical figure of the refracting surface.

Thus our formula has at last assumed a very simple form, and is vastly preferable to Dr Smith’s for practice.

This aberration is evidently proportional to the square of the semi-aperture, and to the square of the distance *φ ;* but in order to obtain this simplicity, several quantities were neglected. The assumption of the equality of AX to g2

-r- is the first source of error. A much more accurate *2a*

value of it would have been for it is really

= — If for AX we substitute its approximated value

e” 2cτe≡

—, we should have AX = 5 = — ⅛. To have

2α \_ e2 4a\*—2

2a

2a used this value would not much have complicated the cal­culus ; but it did not occur to us till we had finished the investigation, and it would have required the whole to be changed. The operation in page 149, col. 2, par. 1, is an­other source of error. But these errors are very inconsi­derable when the aperture is moderate. They increase for the most part with an increase of aperture, but not in the proportion of any regular function of it ; so that we cannot improve the formula by any manageable process, and must be contented with it. The errors are precisely the same with those of Dr Smith’s theorem, and indeed with those of any that we have seen, which are not vastly more com­plicated.

As this is to be frequently combined with subsequent operations, we shorten the expression by putting *θ* for ~~"~~~~1~~~~jj~~~~1~~~~3~~~~"I(~~^3 — ^7^^)⅛' Then *φ2θ* will express the aberra­tion of the first refraction from the focal distance of an in­finitely slender pencil ; and now the focal distance of re­fracted rays is *f* = *φ — φ2θ.*

If the incident rays are parallel, *r* becomes infinite, and *i =* —r— *lf ~.* But in this case *k* becomes = 1/*g*, and *m3* 2 *a*

*1 m—*I *mα , , m, a2*

- = , and p = and p *Ô* becomes ; —

p *ma r m—*1 *r (m—*1)≈

m — I 1 ei e5

X —5— X —ϊ × — = 7s 7τ . This is the aber-

*ms a3* 2 (2m — *∖)ma*

ration of extreme parallel rays.

We must now add the refraction of another surface.

*Lemma* 2. If the focal distance AG be changed by a small quantity *Gg,* the focal distance AH will also be chang­ed by a small quantity HA, and we shall have

*m* ∙ AG2 : AH2 = G*g* : H*h*.

Draw M*g*, M*h*, and the perpendiculars *Gi,* H*k*. Then, be­cause the sines of the angles of incidence arc in a constant ratio to the sines of the angles of refraction, and the incre­ments of these small angles arc proportional to the incre­ments of the sines, these increments of the angles are in the same constant ratio. Therefore we have the angle GM*g* to HM*h* as *m* to 1.

Now G*g* : G*i* = AG : AM,

and G*i* : *hk = m ∙* AG : HA,

and *hk* : H*h* = MA : AH ;

therefore *Gg* : H*h* = *m ∙* AG2 : AH2.

The easiest and most perspicuous method for obtaining the aberration of rays twice refracted, will be to consider the first refraction as not having any aberration, and deter­mine the aberration of the second refraction. Then con­

ceive the focus of the first refraction as shifted by the aber­ration. This will produce a change in the focal distance of the second refraction, which may be determined by this Lemma.

Prop. II. Let AM, BN (fig. 10) be two spherical sur­faces, including a refracting substance, and having their centres C and *c* in the line AG. Let the ray *a*A pass through the centres, which it will do without refraction. Let another ray *m*M, tending to G, be refracted by the first surface into MH, cutting the second surface in N, where it is farther refracted into NI. It is required to determine the focal distance BI.

It is plain that the sine of incidence on the second sur­face is to the sine of refraction into the surrounding air as 1 to *m.* Also BI may be determined in relation to BH, by means of BH, N*x*, Be, and —, in the same way that *m*

AH was determined in relation to AG, by means of AG, MX, AC, and *m.*

Let the radius of the second surface be *b,* and let *e* still express the semi-aperture (because it hardly differs from N*x*). Also let *α* be the thickness of the lens. Then ob­serve, that the focal distance of the rays refracted by the first surface (neglecting the thickness of the lens and the aberration of the first surface), is the distance of the radiant point for the second refraction, or is the focal distance of rays incident on the second surface. In place of *r* there­fore we must take *φ* ; and as we made *k* = 1/*a* - 1/*r*, in order to abbreviate the calculus, let us now make *l* =1/*b -* 1/*φ*; and make 1/*f* = 1/*b* - *ml,* as we made 1/φ = 1/*a - k*/*m.* Lastly, in place of *i — — - j* (⅜\* — ~)77> π>akc — (- l)m3

*le—*q⅛ = - *p -*

*∖ nιφj 2 rn ∖ ρ Hi.*

Thus we have got an expression similar to the other ; and the focal distance BI, after two refractions, becomes BI = *f-f2* θ'*.*

But this is on the supposition that BH is equal to p, whereas it is really φ—φ2 θ*—α.* This must occasion a change in the value just now obtained of BI. The source of the change is twofold. 1st, Because in the value 1/*b -* 1/*φ*, we must put 1/*b* - 1/\*\*\*\*\*, and because we must do the

o p — *pj— a*

*rnt P*

same in the fraction ~~∙ the second place, when the value of BH is diminished by the quantity *φ2θ* + α, BI will suffer a change in the proportion determined by the second Lemma. The first difference may safely be neglected, be­cause the value of *θ*' is very small, by reason of the co-efficient — being very small, and also because the variation bears a very small ratio to the quantity itself, when the true value of p differs but little from that of the quantity for which it is employed. The chief change in BI is that which is determined by the Lemma. Therefore take from BI the