variation of BH, multiplied by *^^γτ,* which is very nearly

*mfz a,*

— . The product of this multiplication is *mf2 i* 4- ~Γ^ ∙ This being taken from *f* leaves us for the value of BI,

+ <,')∙

In this value *f* is the focal distance of an infinitely slen­der pencil of rays twice refracted by a lens having no thick- *mfi*

ness, *a ——* is the shortening occasioned by the thickness, P

and *f2* (*mθ* + *θ'*) is the effect of the two aberrations arising from the aperture.

ft will be convenient, for several collateral purposes, to exterminate from these formulæ the quantities *k*, *l*, and *φ*. For this purpose, make 1/*n* = 1/*a* - 1/*b*. We have already

and---—— + — ’ and Z = - — - = *a r ’ f> a ma ' mr ’ b φ*

**111 1 xτ - 1 1 .. 1 ,**

-— - J Now for 7 write , and we get

*b a ma mr ba n*

*l = —* ! -. Therefore 7 = 7 — *ml* (by construc-

*nuι mr n J 0*

tion, page 150, Prop. II.) becomes = *— α r 7∣*

\_ rn 1 1 \_ *m—* 1 1

n'r *η n ‘ r'*

This last value of j. (the reciprocal of the focus of a slen- wι 1 1

der pencil twice refracted), viz. + -, is the simplest

that can be imagined, and makes *n* as a substitute for

— — 7 ; a most useful symbol, as we shall frequently find in *α b*

the sequel. It also gives a very simple expression of the focal distance of parallel rays, which we may call the prin­cipal focal distance of the lens, and distinguish it in future by the symbol *p ;* for the expression *y. — m* + be­comes - = - when the incident light is parallel. And

*p η* bi

this gives us another very simple and useful measure of *f ;*

for 7 becomes = - + -. These equations, 7 = — 4- -,

*f p r i f n r,*

— = - -, and 7= - + -, deserve therefore to be made

*P n f p r*

very familiar to the mind.

We may also take notice of another property of *n.* It is half the radius of an isosceles lens, which is equivalent to the lens whose radii are *a* and *b;* for suppose the lens to be isosceles, that is, *a = b;* then *n = -—-.* Now the second *a* α

a is negative if the first be positive, or positive if the first . .∙ τι. *r* 1 1 α ^^h *b* <1 + a 2

**be** negative. Therefore 7 = —— = —= -, and

*a o ar ar a*

**12 α**

-=-> and *n=*. Now the focal distance of this lens *η α* 2

is ———, and so is that of the other, and they are equi­valent.

But, to proceed with our investigation, recollect that we

had i = -—*(h3 — —)* Therefore *m θ= —*

*rn3 ∖ r ∕ 2 m*

*∕k3 lc∖* es . 1 λ *m — If , , mili∖ et*

( -)3. Andi,was = — m3Z3H )-.

\»i r∕ 2 *m ∖ <f J 2*

Therefore *f2* (*mθ* + *θ*'), the aberration (neglecting the thickness of the lens), is

c,, 1 *. . f,m — 1 fk3 h2 mals∖ et*

*j j rn ∖m r φ J 2*

If we now write for *h, l,* and *φ,* their values as deter­mined above, performing all the necessary multiplications, and arrange the terms in such a manner as to collect in one sum the co-efficients of α, *n,* and *r*, we shall find four terms for the value of *mθ,* and ten for the value of *θ'*. The four are destroyed by as many with contrary signs in the value of *θ',* and there remain six terms to express the value of *mθ + θ',* which we shall express by one symbol *q ;* and the equation stands thus :

\_ m— 1 Λn, 2m\* + m m+2 3m5 + rn *4rn* + 4

*- m ∖ n3 an2 α2n rnt am*

. 3rn + 2∖ es

**+ rsn ~) 2'**

The focal distance therefore of rays twice refracted, reckoned from the last surface, or BI, corrected for aber­ration, and for the thickness of the lens, is *f —f1 ~—f2 q,* consisting of three parts, viz. *f,* the focal distance of central rays ; *fl* the correction for the thickness of the lens ; and *f2 q,* the aberration.

The preceding formula appears very complex, but is of very easy management, requiring only the preparation of the simple numbers which form the numerators of the frac­tions included in the parenthesis. When the incident rays are parallel, the terms vanish which have *r* in the denomi­nator, so that only the first three terms are used.

We might here point out the cases which reduce the aberration expressed in the formula last referred to, to no­thing; but as they can scarcely occur in the object-glass of a telescope, we omit it for the present, and proceed to the combination of two or more lenses.

*Lemma* 3. If AG be changed by a small quantity *Gg ;* BI suffers a change It, and *Gg* : I*i* = AG2 : BI2. For it is well known that the small angles GM*g* and IN*i* are equal ; and therefore their subtenses *Gk,* I*n* are proportional to MG, NI, or to AG, AI nearly, when the aperture is mo­derate. Therefore we have (nearly)

*Gk* : I*n* : AG : BI, In : I*i* = AM : BI, *Gg* : *Gk =* AG : AM, Therefore *Gg* : I*i* = AG2 : BI2.

Prop. III. To determine the focal distance of rays re­fracted by two lenses placed near to each other on a com­mon axis.

Let AM, BN (fig. 11) be the surfaces of the first lens, and CO, DP be the surfaces of the second, and let *β* be the thickness of the second lens, and *δ* the interval between them. Let the ra­dius of the anterior surface of the se­cond lens be *at,* and the radius of its posterior surface be *b'.* Let *m'* be to 1 as the sine of incidence to the sine of refraction in the sub­stance of the second lens. Lastly, let *p'* be the principal focal distance of the second lens. Let the extreme or mar­ginal ray meet the axis in L after passing through both