. c(2m-{-l) c(m-p2) , u2(2m'-∣-])

α *ma\* a'*

-ttJ⅛+D+ m≡(3,√+ l)(m-l)+ ~~Μ\*'+!)(?=!)~~ *mfa2 v v m'a'*

u(3m' + 2)(m-l)\*

*m'*

Arrange these terms in order, according as they are fac­tors of -4, -, -ι, or independent quantities. It puts on this form : a→2).4-.c(2πl + i).i- ~~« (≤+j.)~~. \_L *m a- v , a m' «*

... ,. , „ a(3m'+ 2)(m—l)s .

(3m -J-1) (?» — 1) — *u3mfi -* ’A *1- —* o.

Let A be the co-efficient of -.,, B that of -, C that of -i,, *a2 a α'2*

D that of -J, and E the sum of the independent quantity ;

, - , . . c(m-l-2) τι ,. \_ w(m'4-2)

that is, let A be = ——B=≈ c(2m +1), C= ——, *m v ■ ' rn'*

D = √∙i (2m' + 2)- + and E = *cm2*

+ m’ (3m' + 1) (w — 1) — u⅛i — w<3w' + ^<w,—

Our final equation becomes a\_b\_Ç ∏ *a- a a!i a! '*

The co-efficients of this equation, and the independent quantity, are all known, from our knowledge of *m, m', dm, dm' ;* and we are to find the values of α and α', and from them, and *n* = 1, to find the values of *b* and *b'.*

But it is evidently an indeterminate equation, because there are two unknown quantities, so that there may be an infinity of solutions. It must be rendered determinate by means of some other conditions to which it may be sub- iected. These conditions must depend on some other cir­cumstances, which may direct our choice.

One circumstance occurs to us which we think of very great consequence. In the passage of light from one sub­stance to another, there is always a considerable portion reflected from the posterior surface of the first and from the anterior surface of the last ; and this reflection is more copious in proportion to the refraction. This loss of light will therefore be diminished by making the internal sur­faces of the lenses to coincide, that is, by making *b = a'.* This will be attended with another advantage. If we put between the glasses a substance of nearly the same refract­ing power, we shall not only completely prevent this loss of light, but we shall greatly diminish the errors which arise from an imperfect polish of the surfaces. We have tried this, and find the effect very surprising. The lens being polished immediately after the figure has been given it, and while it was almost impervious to light by reason of its roughness, which was still sensible to the naked eye, performed as well as when finished in the finest manner.

This condition, by taking away one refraction, obliges us to increase those which remain, and therefore increases the spherical aberrations. And since our formulæ do not fully remove those (by reason of the small quantities ne­glected in the process), it is uncertain whether this condi­tion be the most eligible. We have, however, no direct argument to the contrary.

Let us see what determination this gives us.

In this case -, = γ- = - — 1. For because - = - — 4 *aba nab*

and *n* = 1, we have 1 + τ = -, and τ = - — 1. There- *b a ba*

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fore = — -f- 1. Therefore, in our final equation,

put i — ? -f- 1 in place of -t, and — 1 in place of

.1∙. l, A —C B-p D —2C ι ,. ι .. r, n

and it becomes 3 — -t-E-∣-D—C=0.

*cr o*

Thus have we arrived at a quadratic equation, where is the unknown quantity. It has the form *pχ-* -f- *qx* -f- *r* = 0, where *p =* A — C, 7 = 2C — B — D, *r* = E -f- D — C and *x =*

*a*

Divide the equation by *p,* and we have *xι + ^x* -J- - = 0. Make *s = -* and *t = -,* and we have *xi* 4- *sx* -4-t=0.

*P P*

This gives us finally i, or *x — —* ⅜ s =±= √∣ s2—*t.*

This value of - is taken from a scale of which the unit is *a*

half the radius of the isosceles lens which is equivalent to the first lens, or has the same focal distance with it. We must then find (on the same scale) the value of *b,* viz. -— 1, which is also the value of α'. Having obtained *at,* we must find *b'* by means of the cquation = and therefore

1 1 1 1 mt. *c 1 1 , l*

*b d* η' **n** *bf cι 1 a*

**+ v— l∙**

Thus is our object-glass constructed ; and we must de­termine its focal distance, or its reciprocal -i∙. This is *= m —* 1 — μ *(m' —* 1).

All these radii and distances are measured on a scale of which *n* is the unit But it is more convenient to mea­sure every thing by the focal distance of the compound ob­ject-glass. This gives us the proportion which all the dis­tances bear to it. Therefore, calling P unity, in order to obtain - on this scale, we have only to state the analogy rn — 1 — w *(m'—* 1) : 1 = - : T and A is the radius of α A

our first surface measured on a scale of which P is the unit.

If, in the formula which expresses the final equation for -, the value of *t* should be positive, and greater than 1/4*s*2*,* the equation has imaginary roots ; and it is not possible, with the glasses employed and the conditions assumed, to correct both the chromatic and spherical aberrations.

If *t* is negative and equal to 1/4s2, the radical part of the value is ≡ 0, and 1/*a* = -1/2s. But if it be negative or po­sitive, but less than 1/4s2, the equation has two real roots, which will give two constructions. That is to be preferred which gives the smallest curvature of the surfaces ; be­cause, since in our formulæ which determine the spherical aberration, some quantities are neglected, these are always greater when a large arch (that is, an arch of many de-