more than that of the crown. But indeed the remaining error is hardly worth our notice.

It is evident to any person conversant with optical dis­cussions that we shall improve the correction of the spheri­cal aberration by diminishing the refractions. If we em­ploy two lenses for producing the convergency of the rays to a real focus, we shall reduce the aberration to 1/4th. Therefore a better achromatic glass will be formed of three lenses, two of which are convex and of crown-glass. The refraction being thus divided between them, the aberrations are lessened. There is no occasion to employ two concave lenses of flint-glass ; there is even an advantage in using one. The aberration being considerable, less of it will serve for correcting the aberration of the crown-glass, and therefore such a form may be selected as has little aberra­tion. Some light is indeed lost by these two additional surfaces ; but this is much more than compensated by the greater apertures which we can venture to give when the curvature of the surface is so much diminished. We pro­ceed therefore to

*The construction of a Triple Achromatic Object-glass.*

It is plain that there are more conditions to be assumed before we can render this a determinate problem, and that the investigation must be more intricate. At the same time it must give us a much greater variety of construc­tions, in consequence of our having more conditions neces­sary for giving the equation this determinate form. Our limits will not allow us to give a full account of all that may be done in this method. We shall therefore content ourselves with giving one case, which will sufficiently point out the method of proceeding. We shall then give the results in some other eligible cases, as rules to artists by which they may construct such glasses.

Let the first and second glasses be of equal curvatures on both sides ; the first being a double convex of crown- glass, and the second a double concave of flint-glass.

Still making *n* the unit of our calculus, we have in the first place *a* = — *b = — a' — b'.* Therefore -,—7-, = — *a o*

(-—γ), or ^z = — - = — L Therefore the equation ™

*dml dm" „, , . η* „ 1 1

4- — -I —= 0becomes *u—*1 d—- = 0,or-— -—1.

*πη' πrι' n" n" u*

Let us call this value u'.

We *have y = m—*1; ^∙≡ — *(m'—* 1); *±i = u'(m—*1);

= + =m —m'+ m'<to~ 1), And if we make *rn, — m* = C, we shall have -p- — — C -f- *u,(rn —* 1). Also -ι- “ rn — 1; = rn — 1 — *(m' —* 1 ) = *m — rn,*

= —C'.

The equality of the two curvatures of each lens gives - = J-. Therefore - — — 7 = — -. = n = 1/2 ; and

*a 2η a b ci b, 2*

1 \_ 1 L\_J\_\_

*b" ~ a" n"~ a” U‘*

Substituting these values in the equation (page 154, col. 2, paragraph 4), we obtain the three formulæ,

1. *cm2-*1/2*c*(2*m*+1)+\*\*\*\*\*

2∙ -*m2*+1/2(2*m*' + 1)- + (3*m*' + *l)(m-l*)

2(m'+l)(m-1) (3m'+ 2) (m — I?

*m’ rn,*

, „ cm” (2m 4- I) , cm, *(m* -I- 2)

3. cm' 3m2 *~~>. ,, ~ I +~~* ~~ι~~~~T~~ *~~-I~~ — cdu"l*

*α ηιa"t*

(3,„ + η + + ^+2)=0.

*' , , , ηια" m*

Now, arrange these quantities according as they are co­efficients of—and of or independent quantities. Let the co-efficient of be A, that of -t be B, and the inde­pendent quantity be C, we have

A = ⅞÷2->, B *= cιP (2m* + 1)-⅛±L∖ and *rn x rn*

C *= end +* + à (2m' + 1) + (Sm' + 1) (m-1.∙)

cc"w'(3m -f- 2) . *m' + 2*

*+ cuκηr* -f- i X—t — *hc(2m* + 1 ) — *mκ √—*

*m* ,ι v , 4/w

\_ ⅜⅛l-O-(3^+2^-1Ζ\_ w

*m' ml ∖ ■ '*

*A* B

Our equation now becomes -sj∙— — -f- C = 0.

This reduced to numbers, by computing the values of the l∙3l2 l-207

co-efficients, is —— — 0∙3257 = 0.

*ari a"*

This, divided by 1·312, gives *s* — — 0·92, and *t = —* 0∙2482, — 1/2 *s* = 0∙46, 1/4 *s2* = 0∙2116, and *√*1/4*s2-t* = ± 0∙6781.

And, finally, *J* = 0∙46 ± 0∙6781.

This has two roots, viz. 0∙2181 and— 1∙1381. The last would give a small radius, and is therefore rejected.

Now, proceeding with this value of and the -ζ∙, we get the other radius *b''*, and then, by means of *u*', we get the other radius which is common to the four surfaces. Then, by ψ∙ = — — *d, we* get the value of P.

The radii being all on the scale of which *u* is the unit, they must be divided by P to obtain their value on the scale which has P for its unit. This will give us *a* = —*b* = *a'* = *b'* = 0∙530, *a''*= 1’215,

*b*" = -0∙3046,

P= 1∙

This is not a very good form, because the last surface has too great curvature.

We thought it worth while to compute the curvatures for a case where the internal surfaces of the lenses coincide, in order to obtain the advantages mentioned on a former occasion. The form is as follows :

The middle lens is a double concave of flint-glass ; the last lens is of crown-glass, and has equal curvatures on both sides. The following table contains the dimensions of the glasses for a variety of focal distances. The first column contains the focal distances in inches ; the second contains the radii of the first surface in inches ; the third contains the radii of the posterior surface of the first lens, and an­terior surface of the seoond ; and the fourth column has the radii of the three remaining surfaces.

P. *a. b, α'. bt, a", b".*

12 9∙25 6∙17 12∙75

24 18∙33 12·25 25∙5

36 27·33 18∙25 3817

48 36∙42 24∙33 50·92