the lens *be ;* and it is possible that the difference may be such that the red and violet rays dispersed at II may be rendered parallel at *b,* or even a little divergent, so as to unite accurately with the red ray at the bottom of the eye. How this may be effected by a proper selection of the places and figures of the lenses, will appear by the following proposition, which we imagine is new, and not inelegant.

Let the compound ray OP (fig. 21) be dispersed by the lens PC, and let PV, PR be its violet and red rays, cut­ting the axis in *g* and G. It is required to place another lens RD in their way, so that the emergent rays Rr, Vr, shall be parallel.

Produce the incident ray OP to Z. The angles ZPR, ZPV, are given (and RPV is nearly = ZPR/27), and the intersections G and *g* with the axis. Let F be the focus of parallel red light coming through the lens RD in the op­posite direction. Then (by the common optical theorem) the perpendicular F*g* will cut PR in such a point *g,* that *g*F will be parallel to the emergent ray Rr, and to V*v*. Therefore if *g*D cut PV in *u*, and *uf* be drawn perpendi­cular to the axis, we shall have (also by the common theo­rem) the point *f* for the focus of violet rays, and DF : D*f* = D*f* : D*u* = 28: 27 nearly, or in a given ratio.

The problem is therefore reduced to this, “ to draw from a point D in the line CG a line D*g,* which shall be cut by the lines PR and PV in the given ratio.”

The following construction naturally offers itself. Make GM : *g*M in the given ratio, and draw MK parallel to P*g*. Through *any* point D of CG draw the straight line PDK, cutting MK in K. Join GK, and draw D*g* parallel to KG. This will solve the problem ; and, drawing *g*F perpendi­cular to the axis, we shall have F for the focus of the lens RD for parallel red rays.

The demonstration is evident ; for MK being parallel to *Pg,* we have GM : *g*M = GK : HK = *g*D : uD = FD*f*D, in the ratio required.

This problem admits of an infinity of solutions, because the point D may be taken anywhere in the line CG. It may therefore be subjected to such conditions as may pro­duce other advantages.

1. It may be restricted by the magnifying power, or by the division which we choose to make of the whole refrac­tion which produces this magnifying power. Thus, if we have resolved to diminish the aberrations by making the two refractions equal, we have determined the angle RrD. Therefore draw GK, making the angle MGK equal to that which the emergent pencil must make with the axis, in or­der to produce this magnifying power. Then draw MK parallel to *Pg,* meeting GK in K. Then draw PK, cutting the axis in D, and D*g* parallel to GK, and *g*F perpendi­cular to the axis. D is the place and DF the focal dis­tance of the eye-glass.

2. Particular circumstances may cause us to fix on a particular place D, and we only want the focal distance. In this case the first construction suffices.

3. We may have determined on a certain focal distance DF, and the place must be determined. In this case let GF : F*g* = 1 : tan. G,

F*g* : *fu —* 1 : *m,* m being = 27/28,

*fu* : *fg* = tan. *g* : 1;

then GF : *fg* = tan. *g : m* tan. G ;

then GF — *fg* : GF tan. *g — m* tan. G : tan. *g,* or *Gg* + F*f :* GF = tan. *g* — *m* tan. G : tan. *g,* and GF = *Gg* + F*f* tan.*g*/tan.*g*-*m*tan.G, and is therefore given, and the place of F is determined ; and since FD is given by supposition, D is determined.

The application of this problem to our purpose is dif­ficult, if we take it in the most general terms ; but the na­ture of the thing makes such limitations that it becomes very easy. In the case of the dispersion of light, the angle GP*g* is so small that MK may be drawn parallel to PG without any sensible error. If the ray OP were parallel to CG, then G would be the focus of the lens PC, and the point M would fall on C ; because the focal distance of red rays is to that of violet rays in the same proportion for every lens, and therefore CG : *Cg* = DF : D*f.* Now, in a telescope which magnifies considerably, the angle at the object-glass is very small, and CG hardly exceeds the fo­cal distance ; and CG is to *Cg* very nearly in the same proportion of 28 to 27. We may therefore draw through C (fig. 22) a line CK parallel to PG ; then draw GK’ per­pendicular to the axis of the lenses, and join PK' ; draw K'BE parallel to CG, cutting PK in B ; draw BHI pa­rallel to GK, cutting GK' in H. Join HD and PK. It is evident that CG is bisected in F', and that K’B = 2FD; also K'H : HG = K’B : BE = CD : DG. Therefore DH is parallel to CK, or to PG. But because PF =F'K', PD is = DB, and IH = HB. Therefore *gD =* HB, and FD = K'B = 2F'D ; and FD is bisected in F'. Therefore CD = CG + FD/2.

That is, in order that the eye-glass RD may correct the dispersion of the field-glass PC, *the distance between them must be equal to the half sum of their focal distances* very nearly. More exactly, *the distance between them must be equal to the half sum of the focal distance of the eye-glass, and the distance at which the field-glass would form an image of the object-glass.* For the point G is the focus to which a ray coming from the centre of the object-glass is refracted by the field-glass.

This is a very simple solution of an important problem. Huyghens’s eye-piece corresponds with it exactly. If indeed the dispersion at P is not entirely produced by the refrac­tion, but perhaps combined with some previous dispersion, the point M (fig. 21) will not coincide with C (fig. 22), and we shall have GC to GM as the natural dispersion at P to the dispersion which really obtains there. This may destroy the equation CD = CG + FD/2. Thus, in a manner rather unexpected, have we freed the eye-glasses from the greatest part of the effect of disper­sion. We may do it entirely by pushing the eye-glass a little nearer to the field-glass. This will render the violet rays a little divergent from the red, so as to produce a perfect picture at the bottom of the eye. But by doing so we have hurt the distinctness of the whole picture, be­cause F is not in the focus of RD. We remedy this by