is, unless the product of the density into the pressure al­ways form a constant quantity, which is extremely absurd. But it is equally clear, that if any other ratio which differs from the first one were assumed to subsist among the dif­ferentials, it must lead to the like absurdity of requiring some of the quantities to be both variable and constant at the same time. Whoever therefore admits the law of Ma­riotte, and the *constancy* in the ratio of the specific heats, has no alternative but to reject the common graduation, and indeed every other which would not make the differentials of heat follow the same proportion as do those of the lo­garithms of the volume and of the pressure, compounded of course with the ratio of *k* to 1. Hence the only function of the integral of equation (A) which can consist with the data is

*q* = A + B (1/k log. *p* — log.g ) (B),

where A and B are constants. This shows plainly that, un­der a constant pressure, air expands in geometrical pro­gression for equal increments of heat, as has been deduced from the same data by a very different process under the article HYGROMETRY, and where it is shown that the value of *k* is most probably 1·3333.

Extravagant as are the inconsistencies which necessarily result from coupling any of the common scales of tempera­ture with the two principles above specified, yet this sin­gular oversight, from its having originated in a work of no less authority than the *Mécanique céleste (*v*.* p. 128), has been implicitly copied into almost every subsequent pro­duction on the same subject. For example, we find it per­vading the very valuable and extensive writings of Baron Poisson, whose recent decease science has now to deplore ; as also those of Navier, and of other eminent French writers, whenever they have occasion to treat on heat or sound. But a most notable instance of the sort is to be found in Mr Lubbock’s recent treatise On the Heat of Vapours, as will presently be noticed.

Since *dg* will vanish and change its sign in the first term of equation (A) just when *dq* in that term does so, it is evi­dent that, although the first term may occasionally take the form of a vanishing fraction, it will never vanish nor change its sign. For a similar reason, the second term of equation (A) will never vanish nor change its sign. But since the sum of the two values of *dq,* regard being had to their signs, will be a measure of the rate at which the air may be gain­ing or losing heat ; so it is only when those two values of *dq* are equal, and with contrary signs destroy each other, making, as it were, the decrement of heat annihilate the increment, that the absolute heat can be constant. In that case, which may imply the sudden compression or dilata­tion of air, equation (A) admits of being greatly simplified ; because both terms being then divisible by the same value of *dq,* we have — dg/g = dp/kp. Hence *k* log.g = log.p + C ; so that if the initial values of g and *p* be called *g*' and *p',* we shall have C = *k* log. *pt —* log. *p' ;* and *k* log. g/g' = log. p/p',

or (g/g')*k* = p/p', as might likewise have been readily deduced from equation (B). But we have taken this other method that it might be clearly seen what a mistake some are in who suppose equation (A) to be restricted to the case in which the absolute heat is constant. For it is now evident that under such a restriction there could have been no use whatever in that equation containing *dq,* or indeed in its having the partial differential form at all. However, through some inadvertency, Mr Lubbock has so seriously miscon­strued the investigations of the foreign mathematicians as to allege that their “ theorems rest upon the condition that

the absolute is constant while the sensible heat varies. This,” says he, “ is the most restricted hypothesis which can be made upon the nature of heat.” Now we presume that if such an able mathematician as Mr Lubbock had only paid a little more attention to the writings of these authors, he would have found that they have restricted themselves to no such hypothesis. For although they have not overlook­ed the particular case just alluded to, in which air may be so suddenly compressed or dilated as to have its tempera­ture altered without the absolute heat having had time either to gain or lose, they have treated the subject in a sufficient­ly general manner to include every other case. Thus it was by means of the variable part of the integral of equa­tion (A) that they intended to express the change of abso­lute heat which may at any time accompany a change in *g,* in *p,* or in both together ; and this they might have done quite consistently, had they not unfortunately modified the

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integral into the form A + B —\*\*\*\*\*, with the view of making it embrace the common theory of temperature. But the most curious thing in the whole is, that Mr Lubbock has, after all, borrowed from them this very formula to be a principal link in his investigation, and then coupled it of new with an additional expression to embrace the theory just men­tioned, as if the latter had not already had a sufficient share in it. See Phil. Magazine for May 1840, p. 439.

Since the constancy in the ratio of the specific heats en­ables us to distinguish between the heat which is absorbed in the enlargement of the volume, under a constant pres­sure, and that which alone raises the temperature ; show­ing the latter to exceed the former in the constant ratio of 1 to *k —* 1; it is evident, that if at any stage of a very ex­tensive increase of temperature, the expansion were to cease, the increment of heat which could add the next degree to the temperature would be the same, no matter what volume the air had by this time acquired ; and that such increment would be equal to the decrement of heat for each succes­sive degree of *a* uniform scale, were the enlarged volume of air now cooled down to the first temperature without suf­fering any contraction ; so that the specific heat is in all cases independent of the magnitude of a constant volume. However, Professor Kelland, without adducing any proof, asserts in his Theory of Heat, that the specific heat is greater with a greater volume, which we now see to be ut­terly incompatible with the constancy in the ratio of the specific heats, a principle which he also admits and em­ploys. The like may be said of his coupling this principle with the common scale of temperature, and also of his as­serting that, in the sudden compression of air, the increase of temperature measured by that scale is proportional to the diminution of the volume ; whereas it is the changes in their logarithms that are proportional, as is shown under our articles Hygrometry and Sound. At first sight, and indeed until some adequate means be used to test the sound­ness of such cases as the preceding, every thing in them may seem to be quite correct. Perhaps opposite errors may have destroyed each other, as sometimes happens from discarding or adopting, at certain stages of the process, quantities which may then seem quite inconsiderable, but which ultimately have sufficient influence to change the character of the final result. It is in this way that in the Theory of Heat (p. 98-95) results have been obtained which it is utterly impossible to deduce from the same data by any legitimate reasoning; as will be found to have been long since shown in the discussion of a similar case in the Edinb. Phil. Journal for July 1827, p. 154. (e. e. e.)

Thermometer, *Differential.* See the preceding article, and Barometer, Climate, Cold, and Meteorology.

THERMOPYLÆ, a narrow pass or defile, between the