moon at the earth’s surface, is to the gravitation towards the earth as 1 to 70 times the square of 601/2, or to 256 217 : and the former disturbing force is to the whole of this as 2 to 601/2 at the point nearest the moon, and the second as 1 to 601/2 at the equatorial plane, and the sum of both, re­duced to the direction of the circumference, where greatest, as 3 to 121, that is, to the whole force of the earth’s gra­vitation, as 1 to 10 334 000; and in a similar manner we find, that the whole disturbing force of the sun is to the weight of the particles as 1 to 25 736 000.” Or, if we call the moon’s horizontal parallax *p,* and substitute 1/*p* for the distance, the whole of the lunar disturbing force in the direction of the surface will be 3/2 · \*\*\*\*\* = 3/140 *p*3*;* or, if *z*  be the moon’s zenith distance from any point of the surface, *f* = 3/70 *p3* sin. cos. *z.*

Theorem B. [F.] The inclination of the surface of an oblong spheroid, slightly elliptical, to that of the inscribed sphere, varies as the sine of twice the distance from the circle of contact ; and a particle resting on any part of it, without friction, may be held in equilibrium by the at­traction of a distant body [situated in the direction of the axis].

If a sphere be inscribed in an oblong spheroid, the ele­vation of the spheroid above the sphere must obviously be proportional, when measured in a direction parallel to the axis of the spheroid, to the ordinate of the sphere, that is, to the sine of the distance from its equator ; and when re­duced to a direction perpendicular to the surface of the sphere, it must be proportional to the square of that sine ; and the tangent of the inclination to the surface of the sphere, which is equal to the fluxion of the elevation di­vided by that of the circumference, must be expressed by twice the continual product of the sine, the cosine, and the ellipticity, or rather the greater elevation *e*, the radius be­ing considered as unity : so that the elevation *e* will also express the tangent of the inclination where it is greatest, since 2 sin. cos. 45° = 1 ; and the inclination will be every­where as the product of the sine and cosine.

If, therefore, the density of the elevated parts be con­sidered as evanescent, and their attraction be neglected, there will be an equilibrium, when the ellipticity is to the radius as the disturbing force to the whole force of gravi­tation ; for each particle situated on the surface will be actuated by a disturbing force tending towards the pole of the spheroid, precisely equal and contrary to that portion of the force of gravitation which urges it in the opposite di­rection down the inclined surface. Hence, if the density of the sea were supposed inconsiderable in comparison with that of the earth, the radius being 20 839 000 feet, the greatest height of a lunar tide in equilibrium would be 2·0166 feet, and that of a solar tide ·8097 : that is, suppos­ing the moon’s horizonal parallax about 57', and her mass 1/70 of that of the earth.

Theorem C. [G.] The disturbing attraction of the thin shell contained between a spheroidical surface and its in­scribed sphere, varies in the same proportion as the incli­nation of the surface, and is to the relative force of gravity depending on that inclination, as three times the density of the shell to five times that of the sphere.

We may imagine the surface of the sphere to be divided by an infinite number of parallel and equidistant circles, beginning from any point at which a gravitating particle is situated, and we may suppose all these circles to be divided by a plane perpendicular to the meridian of the point, and consequently bisecting the equatorial plane of the spheroid: it is obvious, that if the elevations on the opposite sides of

the plane be equal at the corresponding points of each circle, no lateral force will be produced ; but when they are unequal, the excess of the elevated matter on one side above that of the other side will produce a disturbing or lateral force. Now, the elevation being everywhere as the square of the distance *x* from the equatorial plane, we may call it *ex2,* and the difference corresponding to any point of that semicircle which is the nearer to the pole of the sphe­roid, will be *e (x'2 — x''2)* = *e* *(x'+ x") (x'— x'').* But *x'* + *x''* is always twice the distance of the centre of the supposed circle from the equatorial plane ; and the distance of this centre from that of the sphere will be cos. ψ, if ψ be the angular distance of the circle from its pole ; and calling *φ* the distance of this pole from the equatorial plane of the spheroid, the distance in question will be cos. ψ sin. *φ,* and *x' + x" =2* cos. ψ sin. *φ* ; and the difference *x'* —*x''* is twice the actual sine of the arc *θ* in the supposed circle, that is, twice the natural sine, reduced in the ratio of unit to the radius of this circle, which is sin. ψ. reduced again to a direction perpendicular to the equatorial plane ; whence *x'*— *x''* = 2 sin. *θ* sin. ψ cos. *φ* ; and *x'2* — *x''2* = 4 sin. *θ* sin. cos. ψ sin. cos. *φ.* Hence it follows, that, in different , positions of the gravitating particle, the effective elevation

at each point of the surface, similarly situated with respect to it, is as the product of the sine and cosine of its angular distance *φ* from the equatorial plane, the other quantities concerned remaining the same in all positions. But the inclination of the surface of the spheroid, as well as the original disturbing force, varies in the same proportion of the product of the sine and cosine of the distance *φ* ; con­sequently the sum of this disturbing attraction and the ori­ginal force will also vary as the inclination of the surface, and may be in equilibrium with the tendency to descend towards the centre, provided that the ellipticity be duly commensurate to the density of the elevated parts.

Now, in order to find the actual magnitude of the dis­turbing attraction for a shell of given density, we must com­pute the fluent of 4 *e* sin. *θ*d*θ* sin. cos. ψdψ sin. cos. *φ,* re­duced first according to the distance and direction of each particle from the given gravitating particle ; and we must

4

compare the fluent with 4/3π, the attraction of the whole sphere at the distance of the radius, or unity. But for the angle *6,* the portion of the force acting in the common di­rection of sin. *θ* is to the whole attraction at the same dis­tance as sin. *θ* to 1, so that the attractive force of any point of the semicircle will be 4 *e* sin.2 *6* sin. cos. ψ sin. cos. *φ,* and its fluxion will be as sin.2 *θ*d*θ*, of which the fluent is 1/2*θ —* 1/2 sin. cos. *θ,* or when *θ* = 180°, 1/2π, and 1/2π sin. ψ will express the effect of the disturbing attraction of the semi­circles, of which sin. ψ is the radius, reduced to the direc­tion of the middle point, of which the distance is 2 sin. 1/2ψ; the reduction for this distance is as its square to 1 ; and for the direction, as the distance to sin. ψ, together making the ratio of \*\*\*\*\*\*\* — 8.\*"\* , t∙, and the ultimate fluxion of the

8 sin.3 I ∙Φ

force will be 2*eπ* sin. ψ sin. cos. *φ* sin. cos. ψ ≡\*\*\*\*\*\*\*-^!-∖⅜-r τ 8 sin.3 5-4>

dψ = \*\*\*\*\*\*\*\*\*\*2eτ ~~~2 .^^∖~~~~c~~~~θ~~~~ι~~~~∖~~ sin. cos. *φ*dψ ; but sin. ψ = 2 sin.

8 sm.3 ⅛∙ψ

, , 1 , z, . , \*\*\*\*\*\*\*\*8 sin.3 cos.3 A-ψ , \*\*\*\*\*\*\*

cos. *iψ,* and the fraction becomes —-—r^i . ■ cos- *4, —*

\* 8sιn.3⅛∙ψ

cos.3 ∣ψ cos. ψ ≡ cos.3 ∣ψ(cos.l ^∙ψ — sin.2 ⅛∙ψ) = cos.4

— cos.3 ê ψ + cos.4 ∣ψ = 2 cos.4 ∣ψ — cos.3 Jψ. Now, tak­ing the fluent from ψ = 0 to ψ = 180°, we have 2∕,cos.4 ⅛ψ2d∣ψ = ∣,∣, and∕, cos.3 ⅛4 × 2d∣ψ = ∣, the differ-