Sect. III.—*Of the Effects of Resistance in Vibrating Mo­tions, whether Simple or Compound.*

THEOREM G. If d*w* + *Ads* + *Bs*d*s* + D*w*ds = 0, we have *eDs*(*w* + B/D*s* + *AS-B/DD*) = *c*; hl*e* being = 1.

*Scholium.* For the better understanding of the mode of investigation which will be employed in these propositions, it will be proper to premise some remarks on the investi­gation of fluxional equations, by means of multipliers. A person unacquainted with the language of modern mathe­maticians, would naturally understand by a “ criterion of integrability,” some mode of distinguishing an expression that would be integrated, from one that was untractable ; while, in fact, this celebrated criterion relates only to the accidental form in which the expression occurs, and not to its essential nature. If we take, for instance, the well-known case of the fluxion of hl *x/y* = hl*x* — hl*y* *,*we have dx/x - dy/y = ydx-xdy/xy, and making this = 0, we have also *ydx—* xdy = 0; and this expression no longer fulfils the conditions of integrability, until we multiply it again by —, and restore it to its perfect form. The direct in­vestigation of such a multiplier is generally attended by in­superable difficulties; and the best expedient, in practical cases, is to examine the results of the employment of such multipliers as are most likely to be concerned in the pro­blem, with indeterminate co-efficients, and to compare them with the equations proposed. In common cases, the find­ing of fluents, when only one variable quantity is concerned, requires little more than the employment of a table of fluents or integrals such as that of Meier Hirsch ; and the truth of the solution is in general tested at once, for each case, by taking the fluxion of the quantity inserted in the table : but for the separation of different variable quanti­ties, where they are involved with each other, the employ­ment of proper multipliers is one of the most effectual ex­pedients; and it is still more essential to the solution of equations between fluxions of different orders, or their co­efficients. Such equations require in general to be com­pared with some multiple of the exponential quantity *emt,* which affords fluxions of successive orders, that have simple relations to each other, especially when d*t* is consi­dered as constant. The multiples of sin. *Ct,* and cos. *Ct,* are also very useful in such investigations, and for a similar reason ; but the solutions that they afford are commonly less comprehensive than the former, though they are often simpler, and more easily obtained. It is not however ne­cessary that the exponent of the multiplier should flow uniformly, as will appear from the first example of a pro­blem which has been solved by Euler in his Mechanics : the subsequent examples will possess somewhat more of novelty.

*Demonstration.* The fluxion of *eη, (w* 4- *ps* 4- 7) is *eη,* (dw + *pds* 4- *(nw* 4- *nps* 4- *nq)* ds) = *eπ\* dw -∣- (p* -∣- nj) *ds* 4- ndsds 4- *nwds) ;* and comparing with this e"\* *(dw* 4- *Ads* 4- *Bsds* 4- Duals), we have n = *D, np = B,* and *B B* 1 , 1 *A P*

*P-~ ~^∩'* and> last∙y> *P + nq = A, q = ——-t- — AD—Β , . λ . .. π. ∕ B*

*—∩j,~i* consequently the fluxion of e (w + 7) i 4. *~~-jj-∣j~~*—) is equal to nothing, and that quantity is constant, or equal to *c.*

*Example.* Let the given equation be that of a cycloidal pendulum, moving with a resistance proportional to the „ , 1 . dds \_ r, ds2 λ

square of the velocity, or 4- *Bs — D* = 0.

*Scholium 2.* The space s being supposed to begin at the lowest point of the curve, the fluxion ds is negative during the descent on the positive side, and the force dd*s* is con­sequently negative, and equal, when there is no resistance, to *Bs, B* being a positive co-efficient, equivalent, in the case of gravitation, to 2g/l or 32/l, *l* being the length of the pendulum, and *g* the descent of a falling body in the first second. The co-efficient — *D* is negative, because the re­sistance acts in a contrary direction to that of the force *Bs,* as long as s remains positive, and coincides with it on the negative side. But in the return of the pendulum the signs are changed, so that the equation can only be applied to a single vibration ; since the two forces in question oppose each other in the same points of the curve in which they d^2

before agreed, while the square must always remain positive.

*Solution.* If we multiply the given equation by ds, and make the square of the velocity, or eo sc ttr = -^5, we ddç dç2

have ds ≡⅛ 4- *Bsds — D* ^ds = 0 = 4dw4- *Bsds—* di2 dti i 1

*Dwds,* and dιc 4- *2Bsds — 2Dwds* = 0 ; which, compared with the theorem, gives us 0 for *A, 2B* for *B,* and — *2D* for *D ;* and the solution becomes

*ccir,s ;* and if 10 = 0 when s = λ, we have λ 4- +

ce2Z7λ \_ 0j or> ptltting £ λ 4- 22Æ» ~ & & ÷ cβ20λ = °, and *c = -βe~2°λ ; β* being also = if 7 =» 1 ÷

2Z>λ∙ We may also substitute « for λ — *s,* and *ce2v\*,* — — ∕3e^z'l''\* λ∖ will become — — *βe 2Dr,* and *w — B 1 B η — 2Ds B* z∣ — 2-Z9g i ∩∩ ∖

*Ds + 2DD~βe =2DD^~r>fe +*

Now e~ ~n' = 1 — *2Dβ* 4- *2DT<P-*⅜D3<f>4- ⅜Z>4<r, — ... ; and (1 4- 2Z>λ) *e ^ 2°'* = 1 4. 2Dλ — 2Z⅛ — 42‰

4- 2Dstf2 4- 4Z>3λβe — *D%3<P — ..*. ; whence *w =*

*(2Ds — 2D* (λ — <r) 4- 4Diλ<r — 2Ds<r. .. ) = (4ZXλ<r — 2Z)2βa — 4D,λ<r, .. . ) = P (2λ, — 7βs 4. ∣ ¼tf3 — + ∙ ∙ ∙ )∙

*corollary* 1. From this solution we obtain the point at which the velocity is greatest; and, by reversing the equa­tion, we may also find the extent of the vibration. For when *dw =* 0, we have *Bs*d*s = Dw*d*s,* and *Bs = Dw,* which is the obvious expression of the equality of the re-