sistance to the propelling force. Putting the greatest value of *w = x,* and the corresponding value of *\* = ς,* we have *x* = *B , B 1 2Dι ■ t> Γi* 1 *B*

*Dt + 2lΓD+ce =Dc∙ s,nce Bl = Dx’ and 2DD = -ce-o∙=f^* e2\*> ('-\*>; whence 1 = e2O('~λ), *ZL)D γ*

and *y — e2D (λ* ; consequently hl7 — 2D (λ — ς) = hl

(1 4- 2Dλ), and 2Dς = 2Dλ— hl (1 4- 2Dλ), and *ς =* ⅛ (2DW -∣ D>λ\* + ... ) = Dλ> - I D'λ≡ + ...

*B B*

And since x = C> we have x = *∖jpjpy* (2Dλ — hl [l + 2Dλ])∙

*Lemma.* I∙or the reversion of a series, or of a finite equation, if *z = αx* 4- *bxi* 4- cz3 4- ∙ ., we have a? = -2 — *l>* 26i — *ac* hi3 — *5abc* 4- *a2d .*

-√" 4 g— z3 s4 +

α3 α5 a7

1*W — 2↑αbtc* 4- *Ga2bd* + 3aV — a3e r  *-9 z\**

The proof of this well-known formula is the most readily obtained by means of a series with indeterminate co-efficients, such as *x* = *Az* + *Bz2 +...,* which, by actual in­

volution, and by comparison with the proposed series, will give the required values of the co-efficients, as expressed in this Lemma.

*corollary* 2. When *w =* 0, we obtain from its value, 2 1 divided by *Bσ,* the equation 2*λ* = *γσ -* 2/3 *- Dγσ* + 1/3  *D2γσ3—*...;and, by reversing this series, we have \*\*\*\*\* <r= — + s^^+^^...,orff=2K-∣Dλ\* + ...5 the difference ,of the arcs of descent and ascent being 4/3*Dλ2*, and the difference of two successive vibrations 8/3*Dλ2*, when the resistance is very small ; this difference

being also 8/3; so that the displacement of the point of greatest velocity is 3/8 of the difference of the successive vibrations.

*Scholium* 2. If *K* be the value of *w* when *Die* would be equal to the force of gravity, and *DK* = *Bl = 2g,* we have *D = 2G/K* , or *H* being the height from which a body must fall to acquire the velocity *√K,* since *K =* 4*gH*, *D* = 1/2*H*, and 2*D* = 1/*H*.

*Scholium* 3. It is natural to imagine that we might ob­tain the time from the equation expressing the velocity in terms of the space, if we merely expanded the value of

into a new series, by means of the Newtonian theorem ; but the fluents thus obtained for the expression of the time are deficient in convergency ; and a similar difficulty would occur if we expressed *σ* in terms of *w* by reversing the series, and divided its fluxion by *√w.* The ingenuity of Euler has, however, devised a method of avoiding these in­conveniences, by supposing the time to begin at the point where the velocity is a maximum ; and it will be necessary, in this investigation, to follow his steps, with some slight variations.

*corollary* 3. In order to find the time of vibration, we take —i = r,andx — *w= z,* thens = r 4- j, *β=zλ—i—* r, w=2⅛<1-∕e-8ncλ~'~^ + + 0). ««J »

beine - — e *z-* ≤--— r-μ-?.e22>(r-x)e2Dr.

*g - Dt' 2DD D + 2DD ’*

but we have teen that e2β (j—×)= -, and — *z* becomes *2DD* + Dr-2DDe0= DT—2Djb(2Dr + 2nM +>∙+^+...

*ΒDP* = — Br — ∣ *BDc* — 1 *BD'r<*  and

2 2 1

■g = rs + g *Dr3* 4. - Dsr4 4^ ∙ ∙ ∙ 1∏ order to reverse

this series, we must put ~ *= y3,* and r = *sy* 4- *Βyi* 4- *cy3* 4- ... ; and by substituting the powers of this series for those of r in the value of ~~ = *y2=r2* 4- Pr5— qγ, 4- ..., we find a = 1, b = — Jr, c ≡ ∣ ps — ∣Q ... $ and r = *y —\Dlf* 4- 1 Z>y Hence dr = *Ay — ∣ Dydy*

4- I *D'-ylAy— . .* . ; and this fluxion, divided by the velo­city *v ~ t∕(x —* a), will be the fluxion of the time ; or, since d ⅛ = 2ydy and *dy =*

d∕-∕1∙ ds D dz D'

*ν B 2^∖xz-zz)* BB√(x-rr)+6K√B

—-di— . anj t∣le fluent becomes *t — \* arc*

√(x' — *zz)* 2√zf

*2z 2D D^ f* 2’

ν 8in∙ τ - 2B - \*) +6k∕b (\*x arc v 6in∙ τ - √[xz — zi])... ; the value of which, taken from *z ~* 0 to w *2D Di .. .*

s = \*∙18 2 √2 \* 3B√x + S5√⅛ (ix<) ∙ ∙ ∙ If we now make *τ* negative, for the ascent of the pendulum, the co-efficients p, r, ..., b, d,..., will change their signs, and the

\_ . <r 2D *IP*

value of *I* wdl be 5-7-5 4- √x 4- x< ÷ ...,

the sum of both being 4^ + ∙ ∙ ∙ > which is the

4D

time of a complete vibration, and the difference —. √x 4-... ***<JIJ***

the effect of the resistance on the whole time involves, therefore, only the second and the higher powers of the co­efficient of the resistance *D ;* and it also disappears with the arc, as x, the square of the greatest velocity, becomes inconsiderable with respect to the velocity itself, and to the time *τ*/*√B.*

Theorem H. If \*\*\*\*\*4. Λ 4- Us = 0, *dt* being con-