stant, we have e\*",(ds 4- σsdt) *= c ; m* being — ⅜√4 z⅛z √(j∕l∙ — *B),* and *a* = ⅜√4 =p *V(⅛A,∙ — B).*

*Demonstration.* The fluxion of em,(ds 4- α.rd∕) is *emt mt*

(d⅛ 4- <zdsdf -(- (mds 4- owιsd')dz) = *e* (d2s 4- (a 4- *m)* dsdZ 4- *amsdt3) ;* and, comparing this fluxion with the pro­posed equation, we have, for the co-efficients, *a* 4- *m = A,* and *am = Β ;* whence *i- m = A, m2 — Am = — Β,*

m = ⅜ J ± √(∣Aj — *B),* and *a = ⅛A =f ^∕()A2 — IT).*

*Example.* Let the equation proposed be that of a cy­cloidal pendulum, vibrating with a resistance proportional to the velocity ; that is, — 4- *A* 4- *Bs* = 0.

*Scholium* 1. The resistance is here adequately expressed, in all cases, by the term *A*ds/dt so that the equation is per­manently applicable to the successive vibrations. Thus, in the second descent, on the negative side of the vertical line, *lis* being negative, and — s becoming nearer to 0, the fluxion ds is positive, and *A* ⅛ is of a contrary character to *Βs,* as it ought to be.

*Solution.* Since *m* = 1/2*A* ± *√*(1/4*A*2 *— B),* and *a* =1/2*A* ± *√*(1/4*A*2 *— B)*, it is obvions that the two radical quanti­ties will be either possible or imaginary, according as 1/4*A*2 is greater or less than *B.*

*Case* i. If *A2* is greater than 4*B*, the resistance being very considerable, the solution becomes

\*\*\*\*\*\*e4 " ~ - + [P =F - \*)]«) = « :

and the velocity *v —* — J∕~ — ¼∕[1∙Λ2 — ∙B])s —

r∙,c ^-∙it+√(4Λ *B)t.* anj ∣f t∣ιe ve]oc∣ty be supposed to vanish when s = λ, and *t —* 0, we have 0 = ∣√4λ — V(]√42 — B)λ-*c =* ∣∕fλ 4- √(∣Λj--B)λ-c,.

*Corollαrg* 1. Hence it appears that such a pendulum would require an infinite time to descend to the lowest point, since the velocity cannot have a finite value when *s* vanishes, the exponential quantity never beginning nega­tive.

*Scholium* 2. The co-efficient *B* may also be written for \*\*\*\*\*

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an actual pendulum, as measured in English feet, -p or 0λ

y, if we call *g* the descent of a falling body in the first se­cond, which is, however, denoted in the works of some . . , , , τ- , dds 32 „

authors by *⅛g,* or even by *∖g.* If we make 4- y » = 0; when *s — l,* the force becomes such that — d⅛ = 32d<2, and — ⅛ = 32, which is the true velocity generated by such a force in a second of time. Supposing *h* to be the velocity with which the resistance would become equal to the weight, we must have for ^∣p ^∕^ ^t∣z' ∙n order that the force represented by *A* may become equal to that of gravi­ty, and *A* = ψ ; and if Λ be the height from which a bo<ly

*\*\*\*\*\*Jl~ ⅛QQ Q*

must fall to gain the velocity *h,* since *h = -.At = ~ =*

Hence it follows, that when *Λi =* 4B, which is the time of Q \_ I

the possibility of alternate vibrations, '7- = -p, and *h = -l, hl* Ö

the resistance becoming equal to the weight when the body has fallen freely through one eighth of the length of tl>e pendulum.

*case* ii. Supposing now the resistance to be more mode­rate, and *jAi* to be less than *B,* and making *H— ⅛Ai= c1∙,* we shall have √((Λ3 — *B) = √( — cv) ≡ √— 1C;* the solution of the equation, ⅛ 4- *A* 4- *Bs =* 0, will then be d 1 Cl £ ^j) + ⅜Λs =F \*z∙— 1 ) = 0 > whence,

by taking the two different values in succession, and adding *∕ KΛtΓe^-rici+e-^~lc,* together their halves, we obtain d I e, I ~

*— JZlCl yf~lCt*

(ds 4- ∣∕Ls) 4 — √— 1 = 0 ; or,

■ — -∙ .∙z=7c',+ -'ct> ,.λ „„„ √- 1 = -= ⅞ (s + J⅛)

√ΣΓicι -√∑∏c,

, e —β \_ *—i-rt ...*

4 = *cs = ce* .Now the imagi

2√-1

nary exponential quantities, thus combined, are the well- known expressions for the sine and cosine of the arc *ct (Elem. Illustr.* § 358) ; and the last equation may be written thus, cos. *ct* 4- J√is) 4- sin. *ctcs = ce~* ; whence » — — p = *iAs* 4- 8^n' ? *Cs — ———.* This fluent, if dr “ cos. *Ct* cos. *ct*

*r* were made to begin when *v* = 0, would only afford us such expressions as have hitherto been found intractable ; but nothing obliges us to limit the problem to this condi­tion, and it is equally allowable to make the time *t* begin when *v* = 1/2*As,* the corresponding value of s being called 5, then 1/2*As = v =* 1/2*As — c;* consequently *c* = 0. The equa- (l.v sin.

tion will then become \*\*\*\*\*\* — 4- *c—'ct* 4- *ΛAdt =* 0; whence s cos,

hls— hl cos. *Ct — d —iAt* = hl —^-7r, and —½τ = \* cos. *Ct* cos. *Ct*

*^c, i'll, or i \_ cos< (Jι∙ e* ' . ant]ι n)ιen *t* = 0, s —

*e — i ;* consequently '- = cos. *ct ∙ e* cos. *ct* ^l —

*⅛At* 4- *AV — ~ AV* 4- .. .). But since t> = —ds= s *(c — Ct∙ dr + jAdt* ), it follows that »must vanish when- √i

ever *C — Ct* 4- *iA* = 0, or when ta. *ct = —*7π,that is, in cos. *λC*

*j* the first instance, very nearly when *Ct =* ancl 1 =

*— A , ,j AΛ 1 —iAt A A*

and — *JiAt =* and *e* = 1 4- very

nearly ; so that, calling the primitive extent of the arc of vibration « = λ, we have - = cos. *ct* (1+4⅜c)ιcos∙ci