since *α* and y are very small angles, we may assume cos. *α* = 1, sin. α = α, cos. *γ* = 1, sin. *γ = γ,* by which the formula becomes, on substituting — (*β* 4- *γ) for a,*

\*\*\*sin. C -∣- 7 cos. C

- sin. A — *(β* 4- 7) cos. A'

Dividing the numerator of this expression by the denomi­nator, and rejecting the terms in the quotient which con­tain the squares and higher powers of *β* and 7, we get

sin. C *β* cos. A sin. C 7 sin. B c ~ sin. A sin.\* A + sin.\* A,

so that the error of the side *c* is

*β* cos. A sin. C 7 sin.\*B sin.2 A sin.\* A ’

Similarly, the error of the side *b* is found to be

7 cos. A sin. Β *β* sin. C sin.\* A sin.\* A ’

It now remains to determine the conditions under which these expressions have the smallest values. Since α 4- *β* 4- 7 = 0, in every case two of these quantities must have the same sign, and the third the opposite. If therefore *α* and *β* are both positive, 7 is negative ; and if they are both negative, 7 is positive ; so that as 7 must of necessity be either positive or negative, there are only two chances for *α* and *β* having the same sign. In like manner, there are also two chances only for *a* and 7 having the same sign and *β* the opposite, and for *β* and 7 having both the same sign and *α* the opposite. It thus appears, that for any two specified errors, there are only two chances for both having the same sign, and four chances for their having opposite signs ; so that in the above expressions the probability that *β* and 7 are of opposite signs, is twice as great as the pro­bability of both having the same sign. Now if they have opposite signs, both expressions will be diminished if cos. A be positive, that is, if the third angle A be less than 90°, or less than a right angle. With respect to the relation be­tween B and C, which gives the smallest chances of error, it may be assumed as probable that the two errors *β* and 7, though they have opposite signs, will not differ greatly in magnitude. But if they are nearly equal, the two expres­sions for the errors of *b* and c will have the smallest values when B and C are nearly equal. Hence the conditions which afford the greatest probability of the smallest errors are these: 1. That the angle opposite to the measured side be less than a right angle ; and, 2. that the angles adjacent to that side be nearly equal. These conditions will be best ful­filled on the average of a series, by making the triangles as nearly as possible equilateral.

If only two angles B and C have been observed, then there is the same probability of *β* and 7 having the same sign or opposite signs. In this case the chance of smallest error is obtained by making cos. A = ft, that is, when A is a right angle.

In the earlier part of the British survey, the signals made use of were of various kinds, but principally flag-staffs car­rying reverberatory lamps, and furnished with concave cop­per reflectors about nine or ten inches in diameter, well polished and silvered, and enclosed in tin cases, having plates of ground glass in front, to prevent the bad effects of unequal and unsteady light from exposure to the wind. Such signals can be seen at the distance of twenty or twenty-four miles. For more distant stations, Bengal or white lights were fixed in small sockets, supported on a tripod of about five or six feet in height. By means of a plummet, these could be readily placed precisely over the point marking the station. The now well-known Drum­mond light (for the description of which see Mr Drum­mond’s paper in the Phil. Trans, for 1826) was practically applied as a night-signal at some of the stations in Ireland,

and the west of Scotland. But the difficulties and incon­veniences of night-signals are so great, and the observa­tions attended with so much uncertainty from the unsteadi­ness and scintillation of the light, that the practice of ob­serving by night has of late years been abandoned. At pre­sent the signals are usually formed by building a conical pile of stones, in some instances exceeding twenty feet in diameter at the base, over the point which marks the cen­tre of the station. Such signals are of course subject to this great inconvenience, that they can only be seen in favour­able states of the atmosphere; but in all other respects they are found preferable to any others. In some of the late operations where the distances were not very great, a plate of metal was used, having a narrow vertical slit cut in it ; and in this case the signal consists of the line of light pass­ing through the slit, and which can be rendered more bril­liant by means of reflectors suitably disposed. Colonel Col­by has also on some occasions successfully employed the *heliotrope ;* a method recommended by Gauss, and which consists in throwing a beam of solar light upon a distant sta­tion by reflexion from a small plane mirror, as that of a sex­tant or reflecting circle. A description of this ingenious but not very convenient method of forming signals for geo­detical purposes is given in Zach’s *Correspondance Astro­nomique,* vol. V. p. 374.

*Of the Calculation of the Sides of the Triangles.*

In measuring angular distances with the theodolite, the plane of the instrument is carefully adjusted to the horizon, so that the angle observed is the horizontal angle at the station ; and, consequently, no previous reduction is ne­cessary, on account of the three summits of the triangle to be computed being unequally distant from the earth’s centre, as is the case when the angles are measured with a sextant or repeating circle. The practice which was al­ways followed in the Ordnance survey, of placing the centre of the theodolite directly under the centre of the signal, also obviated the necessity of a calculation for reducing the observed angle to the centre of the station. Formulæ for both these reductions will however be found in Figure of the Earth, p. 555.

The three angles of any spherical triangle, as is well known, exceed 180° by a certain quantity, called the *sphe­rical excess,* which is proportional to the area of the tri­angle. Let A, B, C denote the angles of a spherical tri­angle, *r* the radius of the sphere expressed in feet, *π* = 3·14159 the ratio of the circumference to the dia­meter, and S the number of square feet in the surface or area of the triangle, then, by trigonometry,

\*\*\*S \_ ltjθ0 r ⅛.

Let E denote the spherical excess = A + B + C —180°, , \*\*\*\_ S × 180°. , r SX 648000\*

then E = —— tn degrees, or L = ——— express­

ed in seconds. In any triangle which can be measured on the surface of the earth, S is very small in comparison of r2, and therefore E is a very small quantity. (In practice it seldom exceeds four or five seconds, though in some of the large triangles observed in the west of Scotland, whose sides exceed 100 miles, it amounted to thirty or forty seconds.) Hence an approximate value of S will enable us to com­pute E with sufficient precision. For this purpose, there­fore, the triangle may be regarded as a plane one ; and on denoting by *a, b, c,* the number of feet in the sides respec­tively opposite to A, B, C, we shall have for the area, S = ½*ub* sin. C. Substituting this in the formula for the sphe­rical excess, we get, in seconds,

*\*\*\*r, ab* sin. C × 64b000 zl

b - 2T⅛