This formula is deduced on the hypothesis that the tri­angle is on a spherical surface, but it applies also to triangles on the surface of a spheroid ; for it may be demonstrated that the spherical excess is the same for triangles on a spheroid and on a sphere, when the latitudes of the stations and their differences of longitudes are the same.

In order to compute the spherical excess of any triangle, it is necessary to know the value of *r*, the radius of curva­ture of the spherical surface. Now the curvature of the arc joining any two stations on a spheroid varies with the latitude of the stations, and also with the direction of the arc in question in respect of the meridian ; but for the pre­sent purpose it will, in general, be sufficient to assume the value of *r,* which corresponds to the curvature of the meri­dian at the mean latitude of the stations, and even to sup­pose it constant for a whole series of triangles contained be­tween two parallels of latitude not distant more than a few degrees. If, however, the triangles are very large, it may be necessary to compute more accurately ; and in such cases the nearest approximation to the true spherical ex­cess will be found by computing, for the mean latitude of the three stations, the curvature of the meridian, and of the circle perpendicular to the meridian, and taking the mean of the two for the value of *r* ; or, which is nearly the same thing, by computing the radius of the vertical circle which cuts the meridian at an angle of 45° at that mean latitude. As the determination of the curvature of the meridian, the arc perpendicular to the meridian, and the geodetical line making any oblique angle with the meridian, are required for various other purposes connected with the survey as well as the present, we shall here subjoin the formulæ by which they are severally computed from assumed dimensions and ellipticity of the earth.

Let ALP be the arc of the meridian passing through the station L, AC the semidiameter of the equator, CP the semi-axis, LM the normal at L, meeting PC produced in N. Assume α=CP, *b* = AC, and *e* = the ellipticity, or such that *b = a* ( 1 + e), and let *l* be the latitude of L, and R the radius of curvature of the meri­dian at L ; then it is shown in Figure of the Earth, p. 559, that

R = *a* (1 - *e* + 3*e* sin.2*l*) (2).

Let R' be the radius of curvature of the arc perpendicu­lar to the meridian at L ; and it is shown in the same ar­ticle, p. 558, that R' = LN, the normal extended to its in­tersection with the polar axis. Now let *n =* LM, the nor­mal at L ; then, by conic sections, R' : n :: *b2 : α2* ; whence R' = ( l + *e)2n.* But *n = a (* 1 — *e* cos.2*l)* (Figure of the Earth, p. 559) ; therefore, rejecting terms containing the square of *e*, as insignificant, we find

R' = *a* ( 1 + *e* + *e* sin.2 *l*) (3).

To find the curvature of the oblique circle, let *r* be the radius of curvature at the point L of a section of the spheroid containing LN, and making with the meridian an angle = *θ; we* have the following expression found by Euler (Lacroix, *Calcul. Differentiel et Integral,* vol. i. p. 578).

\*\*\*\*\_ RlC

r ~ R sin *° i* -∣- R' cos.2 i’

This last expression may be put under a form more con­venient for calculation. Dividing both terms by *r*', substi­tuting 1 — sin.2 *θ* for cos.’ *6,* and converting the result into a series, all the terms of which after the second may be ne­glected, we get

*r* = R (1 + R'-R/Rsin2θ) (4).

Since in any circle the length of a degree is proportional to the radius (it is found by dividing the radius by the con stant number 57∙29578), if we make M = the length in feet of a degree of the meridian at L, P = the length of a degree of the perpendicular arc, and D = the degree of an arc which makes with the meridian an angle = *θ*, we shall have also

D = M (1 + P-M/Psin2θ) .(5),

which is the expression usually given, and by means of which the length of the oblique degree is found in terms of the degrees of the meridian and perpendicular.

Having computed the spherical excess Ε from approxi­mate values of the lengths of the sides (obtained by sup­posing the triangle a plane one), the sum of the three ob­served angles should be = 180° + E. But as every ob­servation is attended with some degree of uncertainty, the probability is infinitely small that the sum will be precisely equal to this quantity in any case. The difference (which in general will amount to some seconds) is the error of the observed angles; and the next question to be considered is, how should the error be apportioned among the three angles, so that the probability of the result being true may be greater than if any other mode of distribution were adopt­ed ? If no reason exists for supposing that one angle has been determined more accurately than another, the error should of course be equally divided among the three angles ; but in practice this is seldom the case, for it will usually happen that one or other of the angles has been determined by a greater number of observations, or by observations made under more favourable circumstances than the others, and consequently the three determinations are not affected with the same probable errors. In the earlier period of the Ordnance survey, and indeed so far as the published account extends, the apportionment of the error appears to have been made in a manner entirely arbitrary, or at least ac­cording to the observer’s judgment of the relative goodness of the observations ; but this objectionable practice is now abandoned, and a uniform method, founded on the theory of chances, adopted. Suppose several observations to have been made of the same angle, and that the seconds of read­ing are *l*, *l'* *l'',* &c., and let *m* be the average or arithmetical mean of the whole ; then *m — l, m — l', m — l'',* &c., are the errors of the individual observations, and the *weight* of the determination, or of the average *m,* is equal to the square of the number of observations divided by twice the sum of the squares of the errors. See. Probability, vol. xviii. p. 635, No. 145. In this manner the *weight* is found for each angle, and the error of the triangle, that is, the difference between the sum of the three angles (each being the ave­rage of the observed values) and 180° + E, is divided into three parts respectively proportioned to the reciprocal of the weights, which parts form the corrections to be added to or subtracted from the angles to which they respectively correspond. We have then three corrected spherical angles, the sum of which is exactly 180° + E.

The three spherical angles of the triangle being thus determined, the next step in the operation is to compute the lengths of the two remaining sides; one side being always known, either by the measurement of a base, or by the previous com­putation of another triangle. Three different methods of computation have been practised. That which first suggests itself is to transform the side whose length is already known in feet, into an arc of a circle (which is done by comparing it with the radius of the earth), and solving the triangle by the usual formulæ of spherical trigonometry. This was the me­thod followed by Boscovich in his measurement of the Italian arc of meridian ; and it was also practised in some instances by Delambre ; but as it involves a somewhat tedious process of calculation, it is not that which is generally adopted. As the distance of any two stations mutually visible from each other is very small in comparison of the whole circumference of the earth, the chord of the intercepted arc will