differ from the arc itself by a quantity which may be com­puted from the known ratio of the chord to the radius of the earth, but which in general is so small as to be insen­sible. If, therefore, from the observed spherical angles we deduce in each case the corresponding angles formed by the chords, and with these compute the sides by plane tri­gonometry, we shall obtain the chords of the arcs intercept­ed between the stations, and thence the arcs themselves or the true geodetical lines on the spheroid. On this principle all the triangles of the Ordnance survey have been com­puted. A third method, which has been followed in all the recent continental surveys, and which was also adopted by Captain Kater in the verification of General Roy’s triangles, depends on a theorem first demonstrated by Legendre ; namely, that a triangle on a sphere or spheroid, which is small in comparison of the whole spherical surface, differs insensibly from a plane triangle of which the sides are re­spectively equal in length to the sides of the triangle on the sphere, and whose angles are respectively equal to those of the spherical triangle, each diminished by one third of the spherical excess. We shall here give formulæ for the two last methods, with an example of their applica­tion.

To reduce an observed horizontal angle to the angle formed by the chords of the containing sides, let *r* be the radius of the sphere on which the triangle is described ; *a*, *b*, c. the sides (the radius and sides being both expressed in feet) ; and, for the sake of abridging, let

\*\*\*\*\*α *b a c*

*-r = a>r =P>~ = 7∙*

so that *α, β, γ,* represent the sides of a similar triangle on a sphere whose radius = 1. Let A be the horizontal angle opposite the side *a,* and A — *x* the corresponding angle formed by the chords ; the object is to find the correction *x* to be applied to the observed angle A. From a well- known formula of plane trigonometry, we have

\*\*\*\*\*\*\*~ . , . chord2 *β* 4- chord8 *γ —* chord8 α

Cos. (A \_\*) = ,, cl.oπl .j χ t.lluπl y »

or, since chord a = 2 sin. *⅛a,*

Cos. (A-x) = ~~^∙~~~~,~~~~⅞β÷¾∙~~~~,~~~~⅜7-»~~~~in~~~~∙~~~~,~~~~J.Λ~~

' ' 2 sin. ∣ p× sin. ⅛ y

Now cos. (A — *x)* = cos. A cos. *x* -f- sin. A sin.ar; and since *x* is small, cos. *x* = 1, and sin. *x = x* (very nearly) ; therefore

Cos. (A — *x) =* cos. A -f- *x* sin. A,

and consequently

Cos. A 4- z sin. A = ⅛J⅛⅛\*-\*7~ i"⅛..(A). τ 2 sin. I/3 × sin. ⅛ 7 k z

Now in any spherical triangle

\_ . cos. α — cos. Æ cos. *γ*

CoS. A = ∙ ½-; f-

sιn. p sin. *γ*

\_ 1 ~ 2 sin∙2 ⅛ a —(1 —2 s∣n>g ⅛ ff) (1 — 2 sin.2 ⅜ 7)

4 sin. U cos. × sin. J y cos. ] *γ* sin.8 *∖β* 4- sin.81 y— sin/j *a* sin.⅜∕3 sin, ] y

^^^ 2 sin. j jffcos. *β* X sin. .! y cos. J y cos. J *β* cos. J *γ ’*

therefore, by reason of the equation (A),

Γ A — c09' A sin. A sin. £ *β* sin. i y \_

S' - cos. ⅜ *β* cos. I y cos. ⅛ *β* cos. Â y ’

whence we find

*x* sm. A = sin. ∣βsin.∣y — (1 — cos. \* *β* cos. i y) cos. A.

Now the arcs *β* and y being small, we may assume, without sensible error, sin. 4 *β* ≡ J *β,* cos. 4/3=1 — ⅛ *βt ;* whence (making also the like assumption for y), we get x sin. A = ⅛ — — ~~~j^~~~~y~~ cos. A ;

4 b

therefore, since 4 *βγ* = -⅛ (ß + 7)2 —1⅛ (3 —yΛ and ⅜ (β, + 7s) = τ⅛ *(β* + y)’ + ⅛ (3 — r)\*. \* ≡in∙ A = ⅛ (3 + *γ)i (1 — c°s∙ a)* — ⅛ (0 — *rr* (1 + eos- A)» and consequcntly, dividing both sides by sin. A,

∙r = 1⅛ (∕j + 7)a tan∙ ⅜ A — ∙⅛ *(β-γ)t* cot. J A.

In this formula *x* is expressed in parts of the radius = 1 ; butin the applications *x* must be expressed in seconds of arc. Now if R" denote the number of seconds in the radius ( = 206,264"∙8), then 1 : *x* : : R" : seconds in arc *x* ; whence *x* R,' = seconds in *x.* Multiplying therefore the above equa­tion by R", and substituting for *α*, *β,* y, their values, we have ultimately

\*\*\*\*\* = τ⅛ (-^) un∙ ⅛ A ’ R"-1⅜ c"t∙ i A B' (6),

from which formula *x* is found in seconds.

In like manner, the corrections are found for the two other horizontal angles B and C, each being expressed in terms of the angle itself and its containing sides. Let these corrections be respectively *x'* and *x",* then it is evident that *x* + *x'* + *x''* = E, the spherical excess, and consequently, that when the true horizontal angles are respectively di­minished by *x, x',* and *x”*, the sum of the three angles thus reduced will be exactly 180°. These reduced angles are the angles for calculation ; and on computing from them the sides, we obtain the chords of the spherical arcs inter­cepted between the stations, whence the arcs themselves are easily obtained by means of the following formula:

Let *φ* be any small arc in parts of the radius = 1 ; then chord *2φ* = 2 sin. *φ* ; but we have also sin. *φ — φ — 1/6 φ3,* therefore chord *2φ = 2φ —* 1/3 *φ3.* Now, if we assume *2φ.*

\*\*\*\*\*= -, or ® = ; then chord - = - —.-ττ-., and consequcntly

r r 2r *r r* 24rs ’

α = chord α-f-^s (7).

In the last term of this expression the chord may be taken for the arc, and we have therefore this rule for find­ing the arc in terms of the chord. Divide the cube of the chord expressed in feet by twenty-four times the square of the radius (in feet), and add the result to the computed length of the chord ; the sum is the corresponding length of the arc.

We shall now give an example of the application of the preceding formulæ by computing the sides of one of the large triangles connecting the west of Scotland with Ireland, with the requisite data for which we have been obligingly furnished by Colonel Colby. The three stations or angular points are Benlomond in Stirlingshire, Cairns­muir-on-Deugh in Kirkcudbright, and Knocklayd in the county of Antrim ; the longest side, from Benlomond to Knocklayd, exceeding ninety-five miles. The data are as follows

At Benlomond the angle subtended by the stations on Knocklayd and Cairnsmuir-on-Deugh was determined from three observations, giving respectively (we repeat only the seconds) 56° 43' 29"∙97, 27'64, 28"∙72. The mean is *56°* 43' 28"∙58 ; whence the errors are respectively + 1·39, —1∙54, + 0·14; the squares of the errors 1·8321, 2·3716, ·0196; the sum of the squares 4·3233; the weight (the square of the rule of observations divided by twice the sum of the squares of the errors) = 9 ÷ 8·6466 = 1·4041 ; and the reciprocal of the weight ∙96l.

At Cairnsmuir-on-Deugh, one observation gave the angle 79° 42' 28''∙69. The weight is assumed = ∙1, whence the reciprocal of the weight is 10.

At Knocklayd, two observations gave the angles 43° 34' 38"∙36, and 35"∙43. The mean is 43° 34' 36''∙89 ; the er­rors respectively + 1∙47 and — 1∙46 ; the weight found as above = ∙4660 ; and the reciprocal of the weight 2446.

We have therefore the following data for the computa­tion of the triangle :