\*\*\*\*\*\*sin. ∙ψ = *2e* (sin. *l —* sin. *l')* cos. *l,.*

Now, sin. *l —* sin. *l'* = 2 sin. ∣(Z — *I)* cos. ⅜ (Z -f- *lf)* ; and by reason of the smallness of *e* we may put cos. 4 *(l* 4- *l')* cos. *l'* = cos.2 Z ; therefore sin. ∙ψ = 2e sin. λ cos.2 Z, or -ψ = 2e λ cos.2 Z. This is the correction for the spheroidi­city; the corrected difference of latitudes is consequently λ-f-∙ψ = λ(l-∣-2e cos.2 Z). Hence on the spheroid the equation (11) becomes

1 , (Dcos. A D2 sin.2 A tan. *1) . „ r .*

Z—z>=∙(~~g~~~~ι √~~ ~~1~~~~,~~ + -5-∏-—i- >(1-f-2ecos.,Z)...(12).

[R'sm. 1’π 2R'2sm. 1" J √ ∖ ∕

To find the difference of the longitudes of A and B, or the angle at the pole, we have in the above triangle sin. P : sin. PAB : : sin. AB : sin. PB ; that is, since P and AB are small, P : sin. *A : : d* : cos. Z'j whence P = *d* sin. A -÷- cos. *l'* in parts of the radius, or in seconds of arc

p \_ D sin. A fl<n

- R'cos.Z'sin. 1" k ,'

Let B be the azimuth of AB on the horizon of the sta­tion B. To find B we may employ the following property of spherical triangles, which also holds good for triangles on a spheroid, viz. tan. ∣P : cot. ^(PAB -∣- PB A) : : cos. ^(PB — PA) : cos. A(PB + PA) ; that is, since PAB = l8θ, — A, and PBA = B — 180o, and conse­quently PAB -∣- PBA = B — A,

**. .,n \*∖ . ιτ>sin∙** *i(,l* **+ Z,)**

cot.l(B-A) = tan.iP^-7-J.

Now cot. ⅜(B — A) = tan. (90o — ⅜(B —A)) ; and since the difference of the two azimuths, or, which is the same, the sum of the two angles PAB and PBA, is always very nearly equal to 180’, the angle 90° — ^(B — A) is always very- small, and we may therefore substitute the arc for the tan­gent without sensible error. For the same reason we may take ∣P for tan.^P. Making these substitutions, doubling, and transposing, we get

B=lβ0∙ + Λ-P⅛i^ ...(14).

The two formulæ (13) and (14) require no correction on account of the ellipticity ; for with respect to the first, the angle APB at the pole is the same on the spheroid as on the sphere ; and with respect to the second, the correction is so small as to be altogether insensible.

In deducing the above formulæ, it has been supposed that the azimuth at the station A only has been observed. If the direction of the meridian at B has likewise been determined astronomically, then another value of P will be obtained by substituting B and Z for A and *l'* in the equation ( 13) ; and the mean of the results should of course be taken as the correct value.

We shall now apply the preceding formulæ to the com­putation of the latitude, longitude, and azimuth of one of the principal stations of the survey ; and we shall select that of Black Down in Dorsetshire, referring it to the station at Dunnose in the Isle of Wight. The following elements are given in the Survey, vol. ii. p. 88, *et seq.* Distance of Black Down from Dunnose = D =314,307∙5 feet (= 59∙528 miles) ; latitude of Dunnose. = Z = 50° 37' 7"∙3 ; angle at Dunnose between Black Down and the direction of the meridian towards the north (= 180° — A) = 84° 54' 52"·5. From this last angle it is obvious that Black Down lies westward and a little to the north of Dunnose.

Assuming the same elements of the spheroid as before, we have *e* = ∙003324 ; whence we easily find 1 + 2 *e.* cos.2 *i* = 1·00267, the logarithm of which is 0·00116. We have also from the formula (3) the radius of the circle perpen­dicular to the meridian = 20,963,000 feet, in round num­bers ; with this value of R', and the given value of D, the computation of the equation (12) is as follows:

\*\*\*\*\*Log. D = 549735

Log. R' = 7∙32145 Log. (D ÷ R')2 = 6∙35180

Log. sin.e A ≡ 9∙99658 Log.(D-r-R') =8∙17590 Log. tan. Z =008573

Log. cos. A = 8∙94763 Co. log. 2 sin. 1'' = 5∙01340

Co. log. sin. 1" *=z* 5∙31443 Log. ( 1 +2« cos.’ Z) = 0∙00116

Log.(l-f-2ecos.2Z)=0∙001l6

4- 28ft10 ≡ i∙44867 — 274,∙87 = 2’43912

Hence Z — *P =* — 274"∙87 4- 28"-10 = — 246,∙77 (the first term being negative because cos. A is negative), and Z' = Z + 4' 6'∙77. But *l =* 50o 37' 7"∙30 ; tlurcfore Z'=ξ 50o 41' 14"∙07. In the Survey, vol. ii. p. 91, the lati­tude of Black Down is found by a different mode of compu­tation = 50° 41'13,∙8.

For the difference of longitude we have to compute the formula P = D sin. A -÷- R' cos. Z' sin. I" ( 13). Making use of the value of *l'* just found, we have

Log. (D ÷ R')= 8∙ 17590 Log. sin. A = 9∙99829 Co. log. cos. *P* = 0∙ 19822 Co. log. sin. 1” = 5∙31443

4862"∙3 = 3∙68684

Hence P = 1­ 21' 2τ∙3. In the Survey the value of P is found to be lo 2O, 46,∙4. The difference, which amounts to 16'', arises from the different elements of the earth’s figure assumed in the Survey, which give the length of a degree of the circle perpendicular to the meridian at the latitude of Dunnose = 367,038 feet; whereas the length of the de­gree corresponding to the more correct value of the radius assumed in the preceding calculation is only 365,874 feet. It is now known that the differences of the longitudes of the stations on the southern coast of England are all given too small, in consequence of this erroneous assumption of the length of the perpendicular degree.

The longitude of Dunnose west of Greenwich having been found by a previous operation = 1° 11' 36", that of Black Down is consequently 2° 32' 38''∙3.

It now remains to find the azimuth at Black Down, by computing the formula ( 14). Here we have

\*\*\*\*\*Log. sin. ⅛(Z + *P)* (= 50o 39,10"∙68) = 9∙88836 Log. cos.∣(Z,- Z)(= 0o 2' 3"∙38) = 0-00000 Log. P (= 4862''-3) = 3∙68684

lo 2' 4O^∙l2 = 3760"∙l2 = 3 57520

and consequently B = 180o -∣- 950 5, 7^∙50— l0 2'40,∙12, or B = 274o 2' 27"∙38. The observed angle PBA was 940 2' 22,'∙75. Hence the azimuth reckoned from the south towards the west was 274° 2' 22''·75, which differs from the computed value by 4''∙63. It may be remarked, that the method of determining the azimuths adopted in the Survey is attended with some degree of uncertainty. It consists, as has been already stated, in taking the mean of the two angles observed with the theodolite between a flag-staff and the pole-star at its greatest elongation west and east. Now, by reason of the altitude of the pole-star in our latitudes above the terrestrial signal, any error in the adjustment of the cross axis of the theodolite to horizontality produces a greater error in the resulting azimuth. Captain Kater(Phil. Trans. 1828, p. 188) considers the observations of the pole- star for determining the perpendicular degree in our lati­tudes, as wholly unworthy of credit.

*Of the Heights of the Stations, and Refraction.*

In order to complete the explanation of the methods of calculation employed in the Survey, it only remains to show how the relative altitudes of the principal stations have been