TRIGONOMETRY.

1. TRIGONOMETRY is a branch of the mathematics which treats of the relations of the sides and angles of triangles, these being expressed by numbers. It is therefore a theory formed by the union of arithmetic and geometry, and it treats of a particular class of geometrical figures, namely, triangles.

There are two kinds of triangles considered in trigono­metry ; *plane triangles,* and *spherical triangles.* The former are figures on a plane, and the latter on the surface of a sphere ; hence we have *plane trigonometry* and *sphe­rical trigonometry,* the principles of which will be explained in succession. In a triangle there are six things which may be considered, viz. the three sides, and the three angles. The main object of the theory is to give rules by which, when some of these are given, the others may be found.

2. The application of numbers to express the sides and angles of triangles has given rise to several terms which require to be defined before we proceed further.

DEFINITIONS.

I. Since angles at the cen­tres of equal circles have to each other the same ratio as the arcs on which they stand, the arcs may be taken as re­presentatives or *measures* of the angles to which they are proportional. Thus, let ACB be any angle contained by the lines AC, BC: if about C as a centre, with any distance or *radius* AC, a circle ABE be described, the arc AB, inter­cepted between the lines CA, CB, is the measure of the an­gle ACB.

II. The circumference of every circle is supposed to be divided into 360 equal parts called *degrees,* and again each degree is subdivided into 60 equal parts called *mi­nutes,* and each minute into 60 equal parts called *seconds ;* and an angle is said to be of as many degrees, minutes, seconds, &c. as there are degrees, and minutes, and se­conds in its measure. The degrees in any arc or angle are indicated by the character o placed over their num­ber, the minutes by an accent ', and the seconds by two accents ". For example, an angle of 25 degrees 12 mi­nutes and 14 seconds is expressed thus, 25° 12' 14''—A right angle is measured by one fourth part of the circum­ference, or a quadrant, and it contains 90°.

III. The *complement* of an arc or angle is the difference between it and a quadrant, or 90°. Thus, supposing AD to be a quadrant, the arc BD is the complement of the arc AB, and the angle BCD is the complement of the angle ACB.

IV. The *supplement* of an arc or angle is its difference from half the circumference, or 180°. Let ADE be a se­micircle; the arc BDE is the supplement of the arc AB; and the angle BCE is the supplement of the angle ACB.

V. The *sine* of an arc or angle is the perpendicular drawn from one extremity of the arc, on the diameter, that passes through the other extremity. The line BF is the sine of the arc AB, or of the angle ACB.

VI. The *versed sine* is the segment of the diameter be­tween its sine and the arc which measures the angle. The line AF is the versed sine of the arc AB, or the angle ACB, and EF the versed sine of the angle BCE.

VII. The *tangent* is a straight line that touches the arc at one end, and is limited at the other end by a straight line drawn from the centre of the circle through the other extremity of the arc. The line AH is the tangent of the arc AB, or the angle ACB.

VIII. The *secant* is the straight line drawn from the centre to the extremity of the tangent. The line CH is the secant of the arc AB, or angle ACB.

IX. The sine, tangent, and secant, of the complement of any arc or angle, are called by abbreviation its *cosine*, *cotangent,* and *cosecant.* Thus, BG = CF, the sine of the arc BD, is the cosine of the arc AB ; and DK, the tangent of the arc BD, is the cotangent of AB ; and CK, the secant of BD, is the cosecant of AB.

*Trigonometrical Notation.*

The sine of an angle whose measure is A, is expressed thus, *sin. A* ; its cosine by *cos. A* ; its tangent by *tan.* A ; its cotangent by *cot.* A ; its secant by *sec.* A ; its cosecant by *cosec.* A ; its versed sine by *ver. siri.* A ; and its coversed sine by *co. νer. sin.* A.

The radius of a circle, when it is supposed the unit of numbers, requires no symbol : sometimes it is represented by the abbreviation *rad.* and also by the letter R.

3. There are two ways in which the theories of trigono­metry have been established. One is by employing a pro­cess of reasoning purely geometrical, such as has been fol­lowed in our article Geometry. In this way the early writers on the subject constructed their treatises, and it served very well for plane triangles, but was less convenient for spherical triangles, by reason of the intricate diagrams required to represent circles lying in different planes ; and also by the complicated relations of the things to be dis­cussed, which could not easily be expressed by words. The second, which is the more convenient way, is by the sym­bolic reasoning of algebra. The section in Algebra on the Arithmetic of Sines is a specimen of this way of treating the subject ; and it is worthy of remark, that the great extension that has been given to the subject is in no small degree due to the simple and convenient way of representing the sines and tangents, &c. of angles, by the abbreviations, *sin. A, tan.* A, &c. instead of referring to the geometrical lines themselves. As a consequence of this, geometrical construc­tions are avoided, and even diagrams are hardly necessary.

4. In applying the notation and doctrines of modern ana­lysis to trigonometry, we regard the measures of angles, and their sines, tangents, &c. as endowed with the properties of variable quantity ; in particular, we ascribe to them the quality of being *positive* or *negative.* The consequences of this hypothesis have been fully detailed in the *Arithmetic of Sines* (Algebra), to which we refer the reader. By this the properties of triangles are expressed by general for­mulæ, which comprehend all particular cases. This way of considering the numeral measures of trigonometrical lines has greatly simplified spherical trigonometry, and all ap­plications of both trigonometries to astronomy and geodeti­cal inquiries.

*The Trigonometrical Table.*

5. The basis of all trigonometrical calculations is a table which exhibits in parts of the radius the numeral values of the sine, tangent, and secant, of each arc in a series pro­