\*\*\*.. R 4-cos. A 2 cos.\* ⅜ A z √><o D\ i

Now —ö = „a " ■ (Algebra, 249, P), and

(6 + *c)t — a"* ≡ (α + *b* + c) *(b + c —* a).

Put *s* = ⅛(a + *b* + c), then *b + c — a = a + b* -f- c—2a = 2(s — *a)* ; we have now *(b* + c)2 — a2 = 4s(s — a), and, ■ i . ■ cos.' A A *s(s — a) .*

by substituting, —⅜⅝ = and

*be ∙. s(s —* α) = R' : cos.’ ∣A.

\*\*\*Theorem X. In any triangle, the rectangle contained by two sides is to the rectangle contained by the excesses of half its perimeter above these sides, as the square of the radius to the square of the sine of half the angle con­tained by the sides.

Supposing, as in last proposition, that *b* and *c* are the sides about an angle A, and *a* the side opposite to *a,* we have, by Theorem IX.

cos. A \_ *b°* + c\* — α2

R *2bc*

Subtract the first of these equals from — 1, and the se-

26c

cond from --τ- ~ 1, and we have

26c

R — cos. A \_ *2bc —* 7r2 — c' + *α, a- — (b —* c)2

R 26c 20c

.. —cos.A 2sin.s⅜A..

Now g ≡ —~~r~~~~8~~~~8~~ (Algebra, 249, P),

and *a^ — (b —* c)2 = (α + *b —* c) (a — *b* + c).

Put *s =* ∣(a + *b* + c), then *a + b — c = a + b* + c — 2c = 2s — 2c, and a — 0-f-c=a4-Z> + c — *2b = 2s — 2b;* we have now *αt — (b —* c)i = 4(i — *b)* (s — c), and gin∙\* - <\* — 6) (\* — c>

R2 - *be ,*

and *be* : (s — *b) (s — c) =* R, : sin.\* ∣ A.

\*\*\*THEOREM XI. In any triangle, the rectangle contained by half the perimeter, and its excess above the base, is to the rectangle contained by the excesses of half the peri­meter above the two sides, as the square of the radius to the square of the tangent of half the angle contained by the sides.

As in the two preceding theorems, putting *b* and c for the sides about the angle A, and *a* for the opposite side, and s =∣ *(α -∣- b* -∣- c), we have found that

*s (s — a) ∙. be =* cos.2 A : R",

and *be : (s — b) (s — c) =* R' : sin.'∣A ; therefore, *ex tεquαli,*

*s(st-a) : (s — b)(s —* c) = cos.' ⅛A : sin.’ ∣A.

But cos.’ ∣A : sin.’ LA = R' ; tan.' $ A ; therefore *s(s —* α) : (s — *b) (s — c') =* R2 : tan.’ ∣A. Scholium. Since we have

tan.'|A\_(s—*b)(s*—c)\_ 1 (s—*α)(s — b)(s—c)*

R' “ »(\* —α) -(4-θ)'- *s ’*

let us put M to denote the expression —~~π~~~~∖(\* ¾\*∑τ5),~~

**X** *s*

and we have —= ∙ have like expressions

for the angles B, C, and, on the whole,

tan.∣A = — R, tan.⅜B = R, tan JC = — R.

*s — a ∙i s b \* s* c

This seems to be the most convenient formula for find­ing the angles of a triangle when the sides are given.

16. Having laid down principles, we are next to apply

them to trigonometrical calculation. The foundation, as has been already observed, is a table which shows, in parts of the radius, the lengths of the sine, and tangent, and secant, to every minute or smaller division of the quadrant. The most extensive table of this kind is that of G. Joachimus Rhæticus, published in 1596. It gives the sines, &c. to every ten seconds. The invention of logarithms has in a great mea­sure superseded the use of the *natural sines,* as they are called, and, except for particular purposes, the logarithms of the natural sines, called *logarithmic sines,* only are used. Of tables containing these, there are innumerable varieties, according as they are of greater or less extent. The *Arithmetica Logarithmica* of Briggs, published in 1628, by Adrian Viacq, in two folio volumes, is the most extensive. His tables give the logarithms of 100,000 numbers, and lo­garithmic sines and tangents to every ten seconds and to ten places of decimals. Hutton’s Tables give the natural sines, and tangents, and secants ; also their logarithms to every minute and to seven figures. Callet’s *Tables Portatives* give the logarithmic sines and tangents to every ten se­conds ; and Taylor’s Tables, in a large quarto, give the same to every second. For general use, Callet’s are the most convenient. Tables which give the logarithms of num­bers and sines, &c. to six, and even to five figures, are also useful. Of the former, we recommend Tables of six-figure Logarithms, superintended by Richard Farley of the Nau­tical Almanac Office (printed for Longman and Co. 1840) ; and of the latter, Tables of Logarithms, under the superin­tendence of the Society for the Diffusion of Useful Know­ledge (Taylor and Walton, 1839). Those who perform many calculations find it convenient to have tables of dif­ferent extent. The size of our work, and the form of its pages, make it quite unsuitable for a useful logarithmic table, which should be as portable as it can be made. Ac­cordingly we have given none.

17. It is customary to treat first of right-angled triangles, and next of such as have all their angles oblique.

RIGHT-ANGLED PLANE TRIANGLES.

These form four distinct cases. Adhering to the nota­tion premised in article 15, we shall denote the hypotenuse by the letter *b,* the sides about the right angle by *a* and c, and the opposite angles by A and C. (See fig. 2.)

\*\*\*Case I. Given the side *a,* and either of the acute angles A, C, and consequently (Geometry, 24, 1) both ; to find the hypotenuse *b* and remaining side *c.*

*Solution.* R : tan. C = side *a* : side *c ;*

R : sec. C = side *a* : hyp. *b* : or cos. C : R = side *a* : hyp. *b.*

Case II. Given the hypotenuse *b,* and either of the acute angles A, C ; to find the sides *a,* c about the right angle B.

*Solution.* R : cos. C = hyp. *b* : side *a,*

R : sin. C = hyp. *b* : side c.

Case III. Given the sides *a*, c about the right angle B ; to find the angles A, C, and hypotenuse *b.*

*Solution.* Side *a* : side *c* = R : tan. C,

cos. C : R = side *a* : hyp. *b.*

The hypotenuse C may be found independently of the angles by the formula *b =* √(n' -f- c') (Geometry, 13, 4). The angle A is found from C, as above.

Case IV. Given the hypotenuse *b,* and a side *u;* to find the angles A, C, and side c.

*Solution.* Hyp. *b* : side *a* = R : cos. C ;

or thus, *b* 4- *a : b — a* = R' : tan.’ ∣C ; then side c = √ ((δ -f- α) *(b—* β)].

The advantage of this solution is, that the same loga­rithms, viz. of *b* -f- *α* and *b — a,* serve to find the side c and angle C.