\*\*\*18. Examples of the solution of right-angled triangles.

Case I. Given-f on⅛θp - r√1V44∕ Required side c

1 angle Ç = 55 44, , and11 t. 6.

and therefore angle A = 34 16. J jt

*calculation by Logarithms-*

R 10-00000 Cos. C 9∙75054

Tan C ltl∙16666 R l0∙006(M)

Side *a*  2∙7lO96 Sideα 2∙7l096

12∙87762 12∙71096

Sidec = 754∙4 2∙87762 Hyp.fc = 912∙9 2∙96042

Here we add together the logarithms of the second and third terms of each proportion, and subtract the logarithm of the first from their sum. The remainder is the logarithm of the fourth term, which is the answer.

Case II. Given ( ⅛ \* = ⅛~,12,, I Required sides and therefore angle A = 47 48. ) α anc t'

R 10-00000 R 10-00000

Sin. A 9∙86970 Sin. C 9∙82719

Hyp. *b*  2∙655l4 Hyp. *b*  2∙65514

Side « = 334-9 2∙52484 Sidec= 303∙6 2∙48233

To save figures, the logarithm of radius may be sub­tracted or added mentally.

r, τrτ *r~∙∙* f side *a* = 4681 Required angles A

\*\*\*Case III. G>ven jgin c = 305∣ anc∣1c, and hyp. *b.*

Side α 2∙67025 Cos. C 9∙89683

Side *c*  2∙56229 R 10 00000

R 10-00000 Side« 2∙67025

Tan. C = 370 57' 9∙89204 Hyp. *b* = 593∙5 2∙77342

The hypotenuse *b* may also be found, by extraction of the square root, thus :

5 = √(αs +ca) = √(468\*+3652)= √(3522409) =593∙5.

\*\*\*Case IV. Given ( ⅛P∙ \* = 664Λ r

1 ., r I *b + a* = 978 ? A, C, and side c.

and therefore ∣022fl = 350J

Hyp. *b*  2∙822l7 R 10-00000

Side *a*  2∙49693 Sin. C 9∙94506

R 10-00000 Hyp. *b*  2∙82217

Sin. A = 28o l3∣' 9∙67476 Side c 585∙1 2∙76723

This case may be resolved also thus, by Theor. III.

*b* 4. *a......................* 2∙99034 *b* 4- *a* 2∙99034

*b — a*  2∙54407 *b-α* 2∙54407

R2 20-00000

2)5∙53441

Tan.’ iC 2)l9∙55373

*c=* 585∙1 2∙76720

Tan. I C = 30o 53⅜' 9∙77686

C = 61 46∣

This second solution, although not given in our books of trigonometry, is better than the first, because the two things sought are obtained by the same logarithms, and in­dependently of each other.

No notice is here taken of geometrical constructions of the cases by scales. We however recommend that the stu­dent construct all the cases geometrically, and refer him to the article Navigation for examples and practical di­rections.

OBLIQUE-ANGLED TRIANGLES.

19. The three angles of any plane triangle being equal to two right angles (Geometry, 24, 1), when two of them are given, the third is also given.

There are four cases of ob­lique-angled triangles.

Case I. When a side *a,* and two angles B, C, and conse­quently the third angle A, which is the supplement of B + C, are given ; to find the sides *b, c.*

\*\*\*SolutION. Sin. A : sin. B =α : *bl ,π ...*

Sin. A : sin. C=a : c∫ 11,∙ \* ' ∙

Case II. Two sides *b, c,* and an angle C opposite to one of them, are given ; to find the other two angles A, B, and the remaining side *a.* (Fig. 7.)

SolutION. *Side c : side b — sin. C : sin.* B (Th. IV.) When B is found, A = 180° — (B 4- C) is known; and sin. C : sin. A ≡ side c : side *a.*

Since any two angles, which make together two right angles, have the same sine, and the angle B in this case is to be found by its sine only being known ; it will have two distinct values, viz. B, and B'= 180°—B. The angle A may also have two values, A = 180°—(B + C), and A'= 180° — (B' + C). Thus there will be two triangles ABC, AB'C, which alike satisfy the conditions of the data. It must however be considered that the three angles of a triangle cannot exceed two right angles, therefore if C be an obtuse angle, B can only be an acute angle, because a triangle cannot have two obtuse angles. Farther, if the side c op­posite to the given angle C be greater than *b,* in this case B can have but one value ; for when *c* is greater than *b,* then C is greater than either of the values of B, one of which is 180 — B (Geometry, 13, 1), and C + B will be greater than 180°, which is impossible (24, 1).

\*\*\*Because 7- = -7—⅛, and sin. B cannot exceed the radius, *b* sin. B

therefore cτ cannot be less than -∙⅛∙ - ; and unless the data ***b .*** it

satisfy this condition, the solution will be impossible.

Case II. Two sides *b, c,* and the angle A between them, are given ; to find the angles B, C, and the remaining side *a.* V

\*\*\*SolutioN. When A is given, B + C its supplement is given; then *b* 4- *c ∙.b—c —* tan. ⅜(B + C) : tan. <j(B— C). (Theorem V.)

Thus A(B — C) becomes known, and B = ∣(B -∣-C) 4- £(B — C), also C = ∣(B + C) — J(B — C), are known. All the angles of the triangle being known, and two of its sides ; the remaining side may be found in two ways by Case I., or better by either of these proportions,

Cos. ⅛(B — C) : cos. ⅛(B -∣- C) = *b + c∙. a,* l y∣ yj Sin.⅛(B- C)rεin.4(B + C) *=b — c-.a.f lheor∙ Vk* If both be used, the τesults will verify each other. The side *a* may also be found by Theorem III.

Case IV. The three sides α, *b, c* are given ; to find the three angles.

\*\*\*SolutioN. Find *s =* ∣(α + *b* 4- c), andM = √(<-

M

Then, by Theorem XL, tan. ⅛A — — — ∙ R, tan. ⅛B

= ∙ R, tan. ⅜C = —— ∙ R.

*9 — b \* 9— c*