Cask IV. Given J side *b =* 5β2 ■- Remnred the angles

(»idee = 320 J A’B’C·

2)1682 Logs.

*s* = i*(a* -f- *b + c)* = 841 7O7Û20 Ar. comp. log. *s — a —* 41 1’61278 s —0 = 279 2∙44560 *s—* c = 521 2∙7l684

2)3∙85042

Μ 1∙92521

R 1000000

RM 11∙92521

— \_ taii<, a 10,3 j2431 A\_64o 1, 55,

RM

-^=rtan.⅛B 9∙l7961⅜B= 16 47 23 j^t = tan.⅜C 9-20837⅜C = 9 10 41

Verification 89 59 59.

Hence the angles come out

A = 128° 8' 50'', B = 33° 34’ 46", C = 18° 21' 22''. The verification shows the correctness of the result.

Various applications of trigonometry have been given in Mensuration and Navigation ; to these we refer the reader for more examples in the calculation of triangles.

Part II.—Spherical Τrιgονομετrυ.

1. In our treatise on Geometry we have defined a *pyramid* (Sect. xi. Def. 6). The most simple solid of this kind is the *triangular* pyramid, or *tetraedron,* contained by four plane triangles, or *faces,* each of which has a side com­mon with the other three. The angles of these form four solid angles. Each of the solid angles presents for our con­sideration six plane angles ; namely, the three angles about the solid angle, and the three angles made by the planes of the faces, which are so related, that any three of them being given, the other three may be found. The investigation of formulæ which furnish rules by which this may be accom­plished, forms the object of what is to follow.

2. In the calculus of sines, the things considered are plane angles. If two of these be inward angles of a triangle, and the third the outward and opposite angle, our formula for the sines, tangents, &c. of the sum and difference of two angles (Algebra) will express relations among the angles of a plane triangle. We are now to investigate formulæ analogous to these, which shall involve the angles of a pyra­mid. Although the calculus which treats of angles made by straight lines in a plane may be established independently of a circle, yet it is conducive to perspicuity to employ arcs of circles as the measures of angles. This has led to the measuring of the angles of a pyramid by arcs of circles on the surface of a sphere whose centre is the vertex ; then the angles made by the planes of its faces will be shown by the angles contained by tangents to the circles at their points of intersection.

3. In treating of spherical trigonometry, we shall begin by explaining some properties of the sphere which depend on principles purely geometrical.

DEFINITIONS.

I. A *sphere* is a solid figure generated by the revolution of a semicircle about a diameter which remains unmoved.

\*\*\*\*\*\*II. The diameter about which the semicircle turns, is the *axis* of the sphere ; and the point which is the middle of that diameter is the *centre* of the sphere.

III. A straight line drawn from the centre to any point in the surface of a sphere, is a *radius* of the sphere. It is evident that all the radii are equal.

THEOREM I. Every section of a sphere made by a plane is a circle.

Let ADBE be a sphere, and DHE the curve line which is the common section of its surface and any plane. From O, the centre of the sphere, draw OF perpendicular to the plane DHE, and draw FH to any point in the curve DHE and join OH. In the triangle OFH, the angle OFH is a right angle ; therefore FH = ^∕(OH- — OF’). Now the lines OH and OF are in­variable for all points in the curve DIIE, therefore FH is invariable, and the curve DHE is the circumference of a circle whose radius is FH.

Def. IV. The common section of a sphere and any plane that passes through its centre, is a great circle of the sphere. Let ADBE be a sphere whose centre is O ; the circle DFE, in which any plane passing through O meets the sphere, is a great circle.

Cor. 1. All great circles of the sphere are equal ; and any two great circles mutually bisect each other.

Cor. 2. A great circle may pass through any two as­signed points on a sphere, which are not the extremities of a diameter. For if straight lines be drawn from the centre to the points, a plane may pass along these lines, and only one plane, whose common section with the sphere will be a great circle.

Def. V. The pole of a great circle is a point on the sur­face of the sphere, from which all straight lines drawn to the circumference of the circle are equal.

Let DEE be a great circle, and O its centre; take D and F any points in its circumference, join OD, OF, and draw OA perpendicular to the plane of the circle, meeting the surface of the sphere in A, and join AD, AF ; because AO is perpendicular to the plane DEE, the angles AOD, AOF are right angles; therefore AD = AF, hence A is the pole of the circle DFE.

Cor. Since a perpendicular to the plane of the circle DFE through its centre must meet the sphere in two points ; a great circle has two poles, A, B, which are the extremi­ties of a diameter of the sphere perpendicular to the plane of the circle, and it can have no more.

Theorem II. The arcs of a great circle between the poles and the circumference of another great circle are qua­di ants.

Let A and B be the poles of a great circle DFE, whose centre is O; let ADB be any other great circle that passes through A and B and meets the circle DFE in E: Because the poles of the circle DEF are at the extremities of AB, the diameter of the sphere that is perpendicular to its plane, the angles ΛOD, BOD are right angles, therefore AD, BD, the arcs of the circle ADB, arc quadrants.

Def. VI. A spherical angle is the angle on the surface