(2.) If an arc be taken negatively, its sine is *negative* in the first half of the circle, but *positive* in the second.

(3.) The cosine of an arc = 0 is unity : it is 0 when the arc = 90° ; between *arc* = 0 and *arc* = 90°, the cosine is *positive,* and between *arc* = 90° and *arc =* 270o the co­sine is *negative,* and between 270° and 360° it is *positive.* A positive and a negative arc have the same cosine, so that to a given cosine the arc may be taken either as *positive* or as *negative.*

(4.) Since tan. A≡ the tangent is *positive* in tl>e

first quadrant of the circle, because the sine and cosine are both positive ; it is *negative* in the second, because the sine is positive and the cosine negative ; it is *positive* in the third quadrant, because the sine and cosine are both nega­tive ; lastly, it is negative in the fourth quadrant, because the sine is negative and the cosine positive. The tangent is = 0 when the arc is 0 or 180°, and *infinite* when the arc is 90° or 270°.

(5.) When the arc is known, we can always give the tan­gent the proper sign. If the arc *X* is not known, and we have simply tan. *x = a,* it cannot be known whether the are be positive or negative ; if *a* be a positive number, the tangent belongs alike to two arcs, *x* and 180° + *x.* If *a* be ne­gative, tan. *a* belongs to an arc between 90° and 180°, or between 270° and 360°. The problem therefore admits of two solutions.

\*\*\*But if tan. *x* =-, and that the two terms of the fraction *n*

are positive, then *x ∙κ* 90o. If *m* be positive and *n* nega­tive, *x ≥..* 90β ; when rn and *n* are both negative, *x ≥\*,* 180° ; lastly, if *m* be negative and *n* positive, *x* 270°.

(6.) The rule of the signs for cotangents is the same as for the tangents, since tan. *x =* ——, or cot. *x = —-—.*

cot. *x* tan.«

(7.) The rule for the signs of secants is the same as that for cosines, seeing that sec. *x=—-—.*

COS. *X*

(8.) The rule for cosecants is the same as for sines, since 1

cosec. *X = -*

sin. *x*

(9.) When an arc is known, the sign of its sine is known ; but when *x,* an unknown arc, is to be found from sin. *x — a,* the sine belongs alike to *x* and 180° — *x,* if *a* be posi­tive ; or to 180° + *x,* and 360° — *x, if a* be negative.

(10.) When an arc *a* is known, the sign of its cosine is known ; but if we have cos. *x = b,* we do not know whether *x* be in the first or fourth quadrant if *x* is positive, or whether it be in the second or third when *x* is negative. The problem therefore has also, in this case, two solutions. The nature of the problem may however determine that which the algebraic rule leaves ambiguous. Delambre, in his Astronomy, says, “ These rules are general, and easily remembered, and never mislead a calculator ; while, on the contrary’, the rules with which most writers have compli­cated trigonometry, are so perplexing that there is hardly an author that has not given false ones.” The rule of the signs was long neglected by astronomers, on the known principle that improvements come slowly into general use.

6. \*\*\*\*\*Let ABC be a spherical triangle, and let O be the centre of the sphere ; draw the radii OA, OB, OC. The solid angle at O is contained by the three plane angles AOB, BOC, AOC, and the measures are the arcs AB, BC, AC, which are the sides of the triangle ABC.

At the point A in the plane AOB, draw AD touching the arc AB, and meeting the plane BOC in D; and in the plane AOC draw AE touching the arc AC, and meeting the plane BOC in E. The angle DAE contained by the tangents AD, AE, is the same as the spherical angle BAC (Def. vi.). Join the points D, E by a straight line, which will be at once the base of the plane triangles DAE, DOE, the vertices of which are joined by the line AO, a radius of the sphere. We shall throughout make this line the *unit,* and express it by 1. Let us now denote the sides and angles of the spherical triangle ABC as follows.

The sides BC *= a, AC — b,* AB = *c ;* The angles BAC = A, ABC = B, ACB = C. We have now

AE = tan. *b — --*~~p~~ OE = scc. *b = .,*

cos. *b* cos. *o*

. τ∙x s∙n∙ c ΛΓ, 1

AD = tan. *c —* , OD = sec. c = .

cos. *c* cos. c

By plane trigonometry (Tn. VIII.), in the plane triangle ADE,

DE2 *f=* AE2 + AD2 — 2 AE∙AD cos. A, ( = tan.2 *b* 4- tan.2 *c —* 2 tan. *b* tan. *c* cos. A ; and in the triangle ODE, we have also

de, f = OE- + OD1 — 2 OE∙OD cos. α,

I = sec.8 *b* -f- sec.2 *c —* 2 sec. *b* sec. *c* cos. a. Hence, subtracting the sides of the first of these equa­tions from the sides of the second, and considering that sec.2 *b—* tan.2 *b* = 1, sec.8 c — tan.2 *c —* 1, we have 0=l-∣-l — 2 sec. *b* sec. c cos. *a* + 2 tan. *b* tan. *c* cos. A ; and hence, again dividing the terms by 2, &c.

sec. *b* sec. *c* cos. *a* = I -f- tan. *b* tan. *c* cos. A ;

, . cos. *a* , 1 sin. *b* sin. *c ,*

that is, ; =1-1 r cos. A ;

cos. *b* cos. c cos. *o* cos. *c*

and multiplying both sides by cos. *b* cos. *c,*

cos. *a =* cos. *b* cos. *c* -∣- sin. *b* sin. c cos. A.

This is the formula given by Bertrand, and it comprehends in itself the whole principles of spherical trigonometry.

7. Since the same property must belong alike to the three sides, we may interchange the letters which represent them and the angles, and thus have

I.

Cos. *a* = cos. *b* cos. *c* + sin. *b* sin. *c* cos. A...(l) Cos. *b* = cos. *a* cos. c 4- sin. *a* sin. *c* cos. B...(2) Cos. c = cos. *a* cos. *b* + sin. *a* sin. *b* cos. C...(3)

8. To deduce other formulæ from these, w∙e add and subtract the first and second, and thus get

, , f (cos. *b* + cos. α) cos. *c*

cos. > -J- cos.a — i + ζsin, α cos, g sin. ⅛ c0s. A) sin. *c,*

, f — (cos. *b —* cos. a) cos. *c*

*cos. b* cos. a — I (sjn, a cos. β — sin. *b* cos. A) sin. *c∙,*

and from these again, by transposing the first term on the right-hand side,

, . , , ,. , Γ sin. *a* cos. B I .

(cos. *b* + cos. α) (1 - cos. c) = ∣ gin b cθg a j sin. *e,*

*, i ∖ .* ∖ I sin. « cos. B I . \_.

(cos. *b* — cos. *a)* ( 1 + cos.c) = ∣ \_ gin b cog a j sin. *c,*

and taking the products of the corresponding sides, and considering that 1 —cos? c = sin.1 *c,* we have, after divid­ing by sin? *c,*

cos? *b —* cos.2 *a =* sin.2 *a* cos.’ B — sin.- *b* cos? A.

Now, since cos.2 *b* -f- sin.2 *b =* cos.2 *a* -f- sin.1 *a,* we have cos.1 *b —* cos.1 *a =* sin.s *α —* sin? *b ;*

therefore, sin.s *a —* sin? *b* =sin.2 *a* cos.1 B — sin.\*' *b* cos.1 A ;