and sin\* *a* (1 —cos.t B) ≡ sin.t Z> (1 —cos.' A); that is, sin.s *a* sin.' B = sin.\* *b* sin.2 A, and sin. *a* sin. B = sin. *b* sin. A.

From this last, we form these three formulæ,

II.

Sin. Z>\_sin. B

Sin. *a ~* sin. A ' )

Sin. c\_sin. C .

Sin. *a* sin. A '

Sin. *b* sin. B

Sin. c - sin. C '

9. By No. 3 of formulæ (I.)

cos. *c —* cos. *a* cos. *b* -p sin. *α* sin. *b* cos. C, therefore cos. *b* cos. c = cos. *a* cos? *b* -f-

sin. α cos. *b* sin. *b* cos. C.

But by No. 1 of formulæ I.

cos. *b* cos *c* = cos. β—sin. *b* sin. *c* cos. A ; therefore cos. *a—* sin. *b* sin. c cos. A = cos. *a* cos.’ *b* -f-

sin. *a* cos. *b* sin. *b* cos. C ;

and cos. a (1 — cos? ⅛) = sin. *b* (sin. *c* cos. A + sin. *a* cos. *b* cos. C).

In this expression put sin.8 *b* for I—cos.2 *b,* and divide both sides by sin. *b* ; the result is

cos. *a* sin. *b z=* sin. c cos. A 4- sin. *a* cos. *b* cos. C....(wι)

But by formula 2 of II. sin. c ≡ s.-~~' r~~ sin. C ; therefore sin. A

, . \_ cos. A . ,

cos. *a* sin. *b =* sin. *α* sin. C .-—i∙ 4- sin. *a* cos. *b* cos. C. sin. A

Divide now by sin. α, and put cot. α for and cot. A for

sin. α

-T—r, and there is obtained sin. A

cot. *a* sin. *b ∑=* cot. A sin. C ψ cos. *b* cos. C. By changing *b* into *e* and C into B, we have also

cot. *a* sin. *c* = cot. A sin. B -f- cos. *c* cos. B.

By treating these two formulæ like those in articles 7 and 8, we obtain a class of six formulæ, viz.

III.

Cot. *a* sin. *b =* cot. A sin. C -f- cos. *b* cos. C (1)

Cot. *a* sin. c= cot. A sin. B -j- cos. *c* cos. B (2)

Cot. *b* sin. *a* = cot. B sin. C + cos. *a* cos. C (3)

Cot. *b* sin. *c* — cot. B sin. A -∣- cos. c cos. A (4)

Cot. *c* sin. *a —* cot. C sin. B -f- cos. *a* cos. B (5)

Cot. c sin. *b —* cot. C sin. A -f- cos. *b* cos. A (6)

In these formulæ, the letters which represent the parts of the triangle are not so symmetrically arranged, as in the two former groups ; and on that account they are less easily remembered. This inconvenience will be obviated by attention to the following remarks.

(1.) Both members of each formula begin with the pro­duct of a cotangent and a sine.

(2.) The first member is formed from any two of the three sides, and the first term of the second member con­tains an angle opposite to one of these, and the angle op­posite to the side which is not in the first member.

(3.) The last term of the second member is formed by the two cosines of the same arcs of which the sines are al­ready in the equation.

10. Resuming equation (*m*) in the preceding article, viz. cos. *α* sin. *b ~* sin. *a* cos. *b* cos. C + sin. c cos. A, change α into *b* and *b* into α, also A into B, and we have sin. *a* cos. *b* = sin. *b* cos. *α* cos. C -f- sin *c* cos. H ; and multiplingboth sides by cos. C, sin.α cos. *b* cos. C = sin. *b* cos. *a* cos.1 C -∣- sin. *c* cos. B cos- C. This value of sin. *a* cos. *b* cos. C being substituted in the former equation, it becomes

cos. *α* sin. *b =* sin. *b* cos. *a* cos., C -f- sin. *c* (cob. B cos. C + cos. A), and by transposing the first term of the second side, cos. *α* sin. *b* ( I —cos? C) = sin. c (cos. B cos. C -f- cos. A ). that is, cos. *a* sin. Λ sin.2 C = sin. *c* (cos. B cos. C -(- cos. A).

Now by (3) of formulæ IL, sin. *b* sin. C = sin. *c* sin. B, therefore this last expression being substituted in the first side of the equation instead of its equal, and both sides be­ing divided by sin. c, we have

cos. *a* sin. B sin. C = cos. B cos. C -f- cos. A.

And by treating the expression just found, as we did those in articles 7, 8, 9, we have these formulæ :

IV.

Cos. A = —cos. B cos. C -J- sin. B sin. C cos. *a* (1)

Cos. B = — cos. A cos. C + sin. A sin. C cos. *b......*(2)

Cos. C = — cos. A cos. B -f- sin. A sin. B cos. c (3)

11. The sides of *a* spherical triangle being still denoted by *a, b, c,* and its angles by A, B, C, let A', B', C', and *a', lit, d,* be arcs or angles, such that

*a* + A' = 180°, *b* + B'= 180°, c⅛C'= 180°; A + α' = 180°, B + *b'* = 180°, C + *d =* 180°.

Then we have

cos. *a = —* cos. A', cos. *b — —* cos. B', cos. c = — cos. C', sin. *α* — sin. A', sin. *b =* sin. B', sin. c = sin. C' ;

cos. A — — cos. α', cos. B = — cos. ∕∕, cos. C = — cos. *d,* sin. A = sin. *a',* sin. B = sin. fr, sin. C — sin. *c.'*

Let these values of the sines and cosines of *a, b, c* and A, B, C be substituted in formulæ I. and IV. ; they then become

cos. A' = — cos. B' cos. C' + sin. B' sin. C' cos. *α',*

cos. B' = — cos. A, cos. C' -J- sin. A' sin. C' cos. *b',*

cos. C' = —cos. A' cos. B' 4- sin. A' sin. B' cos. *d ;*

cos. *a' —* cos. *U* cos. *d* -∣- sin. *b'* sin. *d* cos. A',

cos. *b’ —* cos. *α,* cos. *d* -J- sin. *a'* sin. *d* cos. B',

cos. *d —* cos. a' cos. *b'* -f- sin. *a'* sin. *b'* cos. C'.

Hence, from I. and IV., if the arcs *a*', *b’, d* be taken as the

sides of a spherical triangle, then the arcs A', B', C' must necessarily be the measures of its angles ; thus we have proved analytically our fourth theorem, which has been de­monstrated in article 3, from considerations purely geome­trical. This theorem, which has been known for two centuries past, may be expressed thus: *Any spherical tri­angle may be changed into another of which the sides and angles are respectively the supplements of the angles and sides of the first.* This is the supplement or polar triangle.

12. The four sets of for­mulæ which have been in­vestigated in articles (6), (7), (8), (9) (10), are sufficient for resolving all cases of spherical triangles. When one of the angles is a right angle, they become more simple. We shall now adapt them to that case.

*Right-Angled Spherical Triangles.*

Let us suppose that A — 90°, then cos. A and cot. A are each = 0, and sin. A = 1.

From 1 and 2 of form. I. cos. *a =* cos. *b* cos. *c.*

\*\*\*\*\*Γ . n sin. 6

I sin. B = -—,

From 1 and 2 of II. <(

. \_ sin. c

sin. C =

sin. *α*

From 1 and 2 of III. cot “ sin' b = cos’ k o6ft cot. a sin ∙ = cos. ccos. B.

From 4 and 6 of III. cot∙ b s"κ Γ = C<\*

cot. *e* sin. *b z=* cot. C.