J cos. B cos. C = sin. B sin. C cos. *a,*

cos. B = sin. C cos. *b,*

*( cos. C =* sin. B cos. *c.*

By considering that the cotangent of an arc is the reci­procal of the tangent, and that the tangent is e<pιal to the sine divided by the cosine, we now obtain these ten for­mulæ for right-angled triangles, in which A is the right angle.

V.

Cos. *a =* cos. *b* cos. *c* (1)

Sin. B = ~, sin. C = (2) (3)

sin. *a* sm. *α*

r, ,, tan. c „ tan. *b ... ...*

Cos. B = , cos. C = - (4) (5)

tan.*a* tan.*a ' v*

π, τ> tan. *b* tan. *c . . ea∖*

sin. c sin. *b*

Cos. *a ■=* cot. B cot. C (8)

„ , cos. B cos. C

sιn.C sm. B

13. These ten formulae, which may however be ex­pressed in words by six, are of continual use in astronomy. It is therefore desirable that they should be comprehended in the fewest number possible of practical rules, simple and easily remembered. Such were found by Napier, the inventor of logarithms, and are known by the name of *Napiers Rules of the circular Parts.* They are not new propositions, but are merely enunciations, which, by a parti­cular arrangement and classification of the parts of a tri­angle, include all the six proportions. They are perhaps the happiest example of artificial memory that is known.

In a right-angled spherical triangle ABC, setting aside the right angle A, there remain the sides AC = *b* and AB = *c* about the right angle ; the hypotenuse BC = *a,* and the two oblique angles B, C. The first two of these, viz. the sides *b, c,* and the complements of the remaining three, viz. 90° — a, 90° — B, 90° — C, are called the five *circu­lar* parts. They are called circular, because, when taken in their order, they go round the triangle.

14. When one of the five circular parts is taken as the *middle* part, the two next to it, one on each side, are called the *adjacent* parts; and the other two, each of which is se­parated from the middle by an adjacent part, are called the *opposite* parts.

Thus, if 90° — *a* be the middle part. 90° — C and 90° — B are the adjacent parts, and *b* and c the opposite parts ; if 90° — B be the middle part, 90° — *a* and *c* are the adjacent parts, and 90° — C and *b* are the opposite parts; and if c be the middle part, 90° — B and *b* are the adjacent parts, and 90°—*a* and 90° —C the opposite parts ; and so on.

This arrangement being made, the rule of the circular parts is contained in the following proposition :

*The rectangle under radius and the sine of the middle part is equal to the rectangle under the tangents of the adja­cent parts ; also to the rectangle under the cosines of the op­posite parts.*

It may assist the memory if we represent the middle part by the letter M, the adjacent parts by the letters A, *a*, and the opposite parts by the letters O, *o* ; the rule will stand thus :

\*\*\*\*R∙sin. M = tan. A ∙ tan. *a ;* R ∙ sin. M — cos. O ∙ cos. 0.

15. To verify the rule, let 90o — *a,* the complement of the hypotenuse, be the middle part ; then 90° — C, and 90° — B will be the adjacent, and *b, c* the opposite parts ; and by the rule,

R ∙ sin. (90° — *a) f = tan∙ (90° — b)* tan∙ (9υ° — C)

v 7 I = cos. *ο cos. e ;*

*∏ . λλ, λ f =* cot. B cot. C. This 19 form. (8) ( = cos. *0 cos. c* this is form. (1)

Let 90° — B, the complement of one of the oblique an­gles, be the middle part, then 90° — α and care the adjacent parts, and 90° — C and *b* the opposite. The rule gives R ∙ cos B i = c0t∙ α tan∙ c Fonn’ <4’ 5>

I = sin. C cos. *b* Form. (9, 10)

Let *b,* one of the sides about the right angle, be the middle part, then 90\*— C and ewill be the adjacent parts, and 90° —*a* and 90° — B the opposite parts. In this case, by the rule,

n . , f = cot. C tan. *c..........* Form, f6, 7)

R ∙ sin. *bl \_ . . n t, )*

I = sin. *a* sin. 15 r orm. (2, 3)

These are all the cases of right-angled triangles.

In applying the rule of the circular parts to resolve any case of right-angled triangles, consider which of the three quantities named (the two things given and the one re­quired) must be made the middle term, in order that the other two may be equidistant from it, that is, may be both adjacent or both opposite ; then the one or the other of the theorems which constitute the rule will give the value of the thing required.

Suppose, for example, that c a side about the right angle, and *a* the hypotenuse, are given to find the angle C. It appears that if c be made the middle part, then 9(90° — *a* and 90° — C are the opposite parts ; and therefore that R ∙ sin. *c* = sin. *a* sin. C, and sin. *a* : sin. c = R : sin. C.

16. Delambre has avowed that he found it easier to re­member the six formulæ for right-angled triangles than to apply Napier’s rules. In astronomy, the sun’s longitude ( ) is the hypotenuse of a right-angled triangle, of which the right ascension (*A*R) and declination (D) are the sides about the right angle, the obliquity of the ecliptic (*ω*) is one of the oblique angles. Let the other oblique angle be denoted by A, then the six formulæ for right-angled spherical triangles adapted to astronomical calculation will stand thus :

\*\*\*Sin. D = sin. *ω* sin. Q, tan. .4R = cos. *ω* tan. 0, tan. D = tan. *u* sin. All, cos. o = cos. *ΛR* cos. D, cos. Q — cot. a, cot. A, cos. A = sin. ω cos. ΛR.

The practical astronomer, who has continual occasion to use the first four of these formulæ, will find no difficulty in retaining them in his memory. But Napier’s rules will probably be most easily remembered by students of that science.

17. The general formulæ, I., II., III., IV., serve for all spherical triangles; but they admit of transformations, which make them more convenient for logarithmic calcu­lation.

Resuming the formula, (I.) we put the first under these two forms.