\*\*\*\*\*\*\*\*\*\*Cos.o= (cos.⅛cos. e —sin.6sιn.c)4-sin.6sin.c(l -J- cos.A), Cos.*a=*(cos. *b* cos. c -)- sin.Z>sin.c)—sin.∕>sin.c(l —cos. A).

Now, by the calculus of sines (Algebra, articles 239 and 248),

Cos. *b* cos. c — sin. *b* sin. c = cos. *(b* -f- c),

Cos. *b* cos. c 4- sin. *b* sin. *c* = cos. *(b —* c),

1 + cos. A — 2 cos.2 J A, 1 — cos. A = 2 sin.8 ,∣A.

Hence, by substituting, we get

Cos. *a —* cos. *(b* -∣- c) 4- 2sin. *b* sin. c cos.2 4 A,

Cos. *a —* cos. (Z> — c) — 2 sin. *b* sin. *c* sin.2 J A ;

and transposing, and observing that by the calculus of sines (Algebra, art. 240),

Cos.α — cos. *(b + c)=* 2 sin.] *(a* -⅜- A 4. e)sin.∣ (6 -f-c—o), Cos. (A —c) — cos,α= 2 sin.) (α-∣-c— *b)* sin. j *(a* -∣-⅛ — c), we have

Sin.δsin. ccos.2 jA=sin.A(α 4- *b* -f-c) sin.∣(⅛-f-c—σ), Sin. Λ sin. c sin., ⅛ A = sin. J (a -J- e — *b)* sin. j *(a + l> — c)∙*

To abridge, let us put À (a -f- *b* -f- c) ≡ s ;

then J *(b* 4- *c —* a) *= s — a, ι (a* -j- *c — b) — s — b,* J *(α + b—c) - s — c.*

i. , , . sin. *s* sin. *(s—a)*

>v c have now cos.’ 4 A = rA :

sin.*b* sin.c

sin.’ ⅛a = ⅛4» ⅛L(i=-2).

sin. *b* sin. *c*

There are expressions exactly like these for the squares of the cosines and sines of the halves of the angles B, C ; and from the whole we obtain

Cos.JA = ∕g⅛\*⅞∙i\*-aV

V sιn.o sin. *c*

r, . \_ /sin. *s* sin. (s — *b) z .*

Cos.⅛B = ∕ ——⅞ ' h (α)

¼z sin. asm. *c* j ,

Cos.lC = ∕sA⅛(Lt±) .

\* sin. *a* sin. *b*

Sin.⅛Λ = ∕ginJ⅛.¾gin∙(\*-\*), ’

sin. o sin. c

Sin.]B = L...(tf)

3 √ sin. *α* sin. *c*

Sin.lC= ∕⅛-⅛∙<\*-z,>.

z√ sin. *α* sin. *b*

18. Because tan.\* JA = —therefore

i cos.s⅛A

tan.UA zzsin.(s-⅛)sin.(>-c)

“ sin. *s* sin. (s — *a)*

1 . sin. *(s — a)* sin. *(s — b)* sin. *(s — e)*

= sin.2(s-0) »»i. *s*

*√f* sin.(s—a) sin. (s—⅛)sin.(s—c) l

1 — —8in7Γ2- *f’*

and it is observable that in this expression the sides *a, b, c* enter exactly alike. We have now tan. ∣A — ~. By

putting B and *b,* also C and *c,* successively, instead of A and α, we find like expressions for the tangents of ∣B and ⅜C. Thus we have these three formulæ :

tan. 1A = -——V tan. ⅛B = -.—-. yτ> I

1 sιn.(s- α) 2 sm.(f-*b)* 1

tan. 1C = -—7 ..

sin. *(s— c)* J

The formulæ in this and the preceding article arc ex­actly like those found for the angles of plane triangles. Those for the tangents of half the angles (which we be­lieve to be given in this form for the first time) seem to be most convenient for use in all cases. In using the others, if half the angle sought be nearly a quadrant, its cosine ought to be sought rathu uι\*u ιw uac; ana tne reverse, if the angle be small.

19. We may treat our fourth group of formulæ, of which the first is

Cos. A = — cos. B cos. C + sin. B sin. C cos. σ, in all respects as we have the first, putting it under these forms :

Cos. A = — (cos. B cos. C — sin. B sin. C)

— sin. B sin. C (1 — cos. a),

Cos. A = — (cos. B cos. C -∣- sin. B sin. C)

+ sin. B sin. C ( 1 4- cos. a) ;

from which we have, by the calculus of sines (Algebhλ, art. 239), &c.

Cos. A -∣- cos. (B -(- C) = — sin. B sin. C (1 — cos. a),

Cos. A + cos. (B — C) = sin. B sin. C (1 -f- cos. a). Now

Cos.A 4- cos. (B+C)= 2 cos.,] ( A+B-f-C) cos,⅛(B 4- C—A ), C os. A -∣- cos. ( B—C ) = 2 cos.^( A -f- C—B) cos.( A 4- B—C), 1 — cos. *a — 2* sin.s Jσ, 1 4- cos. *a =* 2 cos.2 ∣a.

Therefore, putting

S=J(A + B + C),

so that S— A = ⅜(B-(-C — A),

S —B= ■ (A + C —B),

S — C= J(A + B —C), we have

Sin. B sin. C sin.’ *⅛a = —* cos. S cos. (S — A),

Sin. B sin. C cos.2 *⅛a =* cos. (S — B) cos. (S — C).

There are like expressions which involve the squares of the sines and cosines of *lb* and lc, from all which we de­duce

, ∕ — cos. S cos. (S — A)'

s,n∙≡α = √ sCT⅛

sin / — cos. S cos. ( S — B) I

V sin. A sin. C

Sin,iβ= /=.?»· S cos-(S-C)

“ V sin. A sin. B

∕cos∙(S—B)cos. (S — C)

Cos. 4*a — ∕* 7 ⅞, .—τ~ »

i ¼∕ sin. B Sin. C

c∙>-ii=√i2h⅛⅛≡⅛=3∙ h« Coslc- ∕co3∙(s~ a)co8∙(S-b)

’ '1 λ< sin. A sin. B ' J

20. Dividing now the cosines of the halves of the sides by their sines, thereby getting expressions for the tangents, and proceeding as in article 18, putting’

∕ (cos.(S—A)cos.(S — B)co$.(S — C)"J

m=√ γ —cos.S i’

we have these formulæ :

(∕)

, ∙ M' ... M'

^-θt∙ .jΛ — ∕n A V Cθt∙ *.)b — r,--* ∕Q T3∖,

i cos. (S — A) cos. (S — B)

, M'

cos.(b — C)

The sum of the three angles of a spherical triangle al­ways exceeds two right angles ; therefore S, half their sum, exceeds a quadrant ; and hence cos. S will be a negative quantity, and — cos. S will be positive, and its square root a real quantity.

In astronomical calculations, it very seldom happens that the angles are wanted, from the three sides being given.

21. Recurring to the formulæ (a), (∕3) of article 17, we deduce from them the following equations.

Sin. ⅛Bcos. |C\_ sin, (s — c)\_ sin. ¾ (a + *b —* c)

cos. IA sin.α - sin. α ’

Sin. ^C cos. ^B sin, (s — ⅛)\_ sin, à (n + c —

cos. ⅛A - sin. *a* sin. *a*