a common side AD. We have (art. 13) tan. AD = tan. C sin. CD = tan. B sin. BD ; hence the angles ACD, ABD, are either both acute or both obtuse. If therefore ACB and ABC be of the same affection, the perpendi­cular falls within the triangle ABC ; but if the angles ACB' and AB'C are of contrary affection, the perpendicular falls without the triangle AB’C.

*Solution of Right-Angled Triangles.*

A spherical triangle may have one right angle, or it may have two, or three. If it have two right angles, the sides opposite to these will be quadrants, and the third the measure of the remaining angle. If it have three right angles, it will be equilateral, and its sides quadrants. We shall set aside these cases, and consider only that in which one of the angles is a right angle, and the other two ob­lique, which may be either acute or obtuse. The side op­posite to the right angle is named the hypotenuse.

The solutions are to be derived from the formulæ in art. 12 ; and as in the investigations, to abridge, the radius was supposed to be the unit, we shall in the rules express it as in the formulæ of plane trigonometry by the letter R.

Case I. Given the hypotenuse *a* and *b*, one of the sides about the right angle, to find the other side *c,* and the oblique angles B, C. These are to be found from formulæ (1), (2, 3), (4, 5).

Solution.

\*\*\*\_ cos. α . sin. *b* tan. *b .*

Cos. c = r R, sin. B = R, cos. C ≡ IL

cos. *b* sin. *α* tan. *n*

If *a* and *b* be of the same affection, then *c* must be less than a quadrant, and also the measure of the angle C. But if *a* and *b* be of different affection, then the side *c* and the angle *c* must be both greater than quadrants. The angle B and the side *b* will be of the same affection.

Case II. Given the sides *b* and *c* about the right angle, to find the hypotenuse *a*, and the angles B, C. Here the formulæ to be used are (1), (6, 7).

Solution.

,, cos. ύ cos. c r, tan. 6 „ \_ tan. *c „*

Cos.*a= „* , tan. B=- R, tan. C = . . R.

R sin. *c* sin. o

In all the cases it must be considered whether the arcs found be less or greater than quadrants, by the algebraic signs + or -, which belong to the given arcs and angles as expressed in article 5.

Case III. Given the hypotenuse a, and an oblique angle B, to find the sides *b, r,* and the other oblique angle C. The formulae to be employed are (2, 3), (4, 5*).*

Solution.

\*\*\*, sin. a sin. B tan. fl cos. B

Sin. *b =* ∩ , tan. c = ∩ ,

cos. *Q \_ tall∙* jjt tan. *a*

Case IV. Given a side *b* and the opposite angle B, to find the hypotenuse *a,* the side *c*, and the anglc C. This is resolved by (2, 3), (6, 7), (9, 10).

Solution.

... sin. *b ,1 .* tan. *b r, . \_* cos. B \_

Sin. *a =* -—∏ R, sm. *c =* ≈ R, sm. C ≡ — R.

sin. B tan. B cos. *b*

In this case each of the things required has two values, because an arc and its supplement have the same sine. Hence two distinct triangles may be found from the same data ; this is called the *ambiguous* case of right-angled spherical triangles.

Case V. Given *a* side *b* with the adjacent angle C, to find the hypotenuse *a*, the side c, and the angle B. These are to be found by (4, 5), (6, 7), (9, 10).

SolutioN.

\*\*\*\*\_ tan. *b n* sin. *b* tan. C

I an. *a ~* — K, tan. *c = ,*

cos. C R

τ, cos. *b* sin. C cos. B = 5——.

**it**

Case VI. Given the two oblique angles B, C, to find the hvpotcnuse *a,* and the sides *b, c.* These are found bv (8), (9, 10).

SolutION.

r, cot. B cot. C . cos. B τ, cos. C „

Cos. a = 1,—-, cos. *b = ——-,* R, cos. *c —* 5 R.

R sm. C sin. B

In addition to what has been said in regard to the arcs or angles sought being less or greater than quadrants, it may be useful to remember, that when the thing to be found by the formulæ is either a tangent or a cosine, and when of the tangents or cosines employed in its computa­tion, one only is an obtuse angle, the angle required is also obtuse. In what follows we shall consider R = 1.

*Oblique-Angled Spherical Triangles.*

24. Before entering on these, we shall resolve this pro­blem.

PROBLEM. Let M, N, P be given quantities, and *x* an unknown arc or angle ; it is proposed to find *x* from this equation :

M cos. *x* + N sin. *x* = P.

\*\*\*\*Let us assume M = *v* cos. *φ,* N ≡t *v* sin. p ; where *v* de­notes a number, and *φ* an angle, both to be determined. Our equation may now be expressed thus.

*v* (cos. *φ* cos. *x* -f- sin. *<p* sin. *x)* = P ;

p

hence *v* cos. *(x — φ) —* P, and cos. *(x —* p) = -.

XT . t>sin. *φ* N ...

*r vcos.φ* M v ,

anj ν \_ \_M N\_

- cos. *φ~* sin. *φ* P P

Therefore cos. (tr∙—p) = — cos. *φ* = — sin. p....(2)

The angle *φ* is determined by the first equation, and the angle (x — *φ)* by the second j therefore *x = (x — φ)* -f∙ p is found.

If we assume M = *v* sin. *φ,* and N = » cos. *φ.* we have t> (sin. *φ* cos. *x* -j- cos. p sin. *x) —* P, and *v* sin. *(x* -f- *φ) —* P;

then cot. p= (1)

P p

and sin. *(x* -j- *φ)* = ~ sin. *φ —* — cos. *φ* (2)

Thus *φ* and *x — (x -∣- φ) — φ* are both determined.

The assumptions tan. *φ* or cot. *φ* are evidently always possible, since an arc *φ* may be found whose tangent or co­tangent shall have any real value whatever.