the path of a reflected sound in the plane of the axis. The last angle of reflection IHA is equal to the last angle of in­cidence FHC. The angle BFH, or its equal CFD, is equal to the angles FHD and FCH ; that is, the angle of inci­dence CFD exceeds the next angle of incidence FHC by the angle FCD, that is, by the angle of the cone. In like manner, FDH exceeds CFD by the same angle FCD. Thus every succeeding angle, either of incidence or reflection, exceeds the next by the angle of the cone. Call the angle of the cone *a*, and let *b* be the first angle of incidence PDC. The second, or DFC, is *b—a.* The third, or FHC, is *b —* 2 a, &c. : and the nth angle of inci­dence or reflection is *b — n* *a,* after *n* reflections. Since the angle di­minishes by equal quantities at each subsequent reflection, it is plain, that whatever be the first angle of incidence, it may be exhausted by this diminution ; namely, when *n* times *a* exceeds or is equal to *b.* Therefore, to know how many re­flections of a sound, whose first incidence has the inclination *b,* can be made in an infinitely extended cone, whose angle is a, divide *b* by *a*; the quotient will give the number *n* of reflections, and the re­mainder, if any, will be the last angle of incidence or reflection less than *a*. It is very plain, that when an angle of reflection IHA is equal to or less than the angle BCA of the cone, the reflected line HI will no more meet with the other side CB of the cone.

We may here observe, that the greatest angle of inci­dence is a right angle, or 90°. This sound would be re­flected back in the same line, and would be incident on the opposite side in an angle = 90° — *a*, &c.

Thus we see that a conical trumpet is well suited for confining the sound ; for, by prolonging it sufficiently, we can keep the lines of reflected sound wholly within the cone. And when it is not carried to such a length as to do this, when it allows the sounding line GH, for example, to escape without farther reflection, the divergency from the axis is less than the last angle of reflection BFH by half the angle BCA of the cone. Let us see what is the connection between the length and the angle of ultimate reflection.

\*\*\*\*\*We have sin. *(b — a)* : sin. *b* = CD : CF, and CF = sin. *b*

CD \* sin. *(b —a)'* aπd sin. (0 — 2 a) : sin. (6—α) = CF : CH, and CH = CF × ⅞,-⅛°> = CD × -≠,"∕ ∖ × ειn.(o—2<∕) sιn.(6—*a)*

*ooltSb-a)* = CDχ ~~. ζ'~~~~n~~~~∙\*~~~~ι~~ , &C.

sin *(b*—2α) sm.(o—*ia)*

Therefore if we suppose X to be the length which will give us *n* reflections, we shall have X = CD × sin.*b*/sin. (*b*-*na*). Hence we see that the length increases as

the angle *(b—na)* diminishes; but is not infinite, unless *n a* is equal to *b.* In this case, the immediately preced­ing angle of reflection must be *a,* because these angles

have the common difference *a.* Therefore the last reflected sound was moving parallel to the opposite side of the cone, and cannot again meet it. But though we cannot assign the length which w ill give the nth reflection, we can give the length which will give the one immediately preceding, whose angle with the side of the cone is *a*. Let Y be this length. We have Y = CD × sin.*b*/sin. *a*. This length will allow every line of sound to be reflected as often, saving once, as if the tube were infinitely long. For suppose a sonorous line to be traced backwards, as if a sound entered the tube in the direction *i h,* and were reflected in the points *h,f, d,* D, the angles will be continually augmented by the con­stant angle *a.* But this augmentation can never go farther than 90° + ½*a.* For if it reaches that value at D, for in­stance, the reflected line DK will be perpendicular to the axis CN ; and the angle ADK will be equal to the angle DKB, and the sound will come out again. This remark is of importance on another account.

Now, suppose the cone to be cut off at D by a plane per­pendicular to the axis, KD will be the diameter of its mouth-piece ; and if we suppose a mouth completely oc­cupying this circle, and every point of the circle to be sonorous, the reflected sounds will proceed from it in the same manner as light would from a flame which completely occupies its area, and is reflected by the inside of the cone. The angle FDA will have the greatest possible sine when it is a right angle, and it never can be greater than ADK, which is = 90° + ½*a.* And since between 90° + ½*a,* and 90° — ½*a*, there must fall some multiple of *a*, call this mul­tiple *b.* Then, in order that every sound may be reflected as often as possible, saving once, we must make the length of it X = CD × sin.*b*/sin.*a*.

Now, since the angle of the cone is never made very great, never exceeding ten or twelve degrees, *b* can never differ from 90° above a degree or two, and its sine cannot differ much from unity. Therefore X will be very nearly equal to , which is also very nearly equal to \*\*\*\*——i— ;

1 sin. *a j* 2sm.⅜α

because *a* is small, and the sines of small arches are nearly equal and proportional to the arches themselves. There is even a small compensation of errors in this formula. For as the sine of 90° is somewhat too large, which would give X too great, 2 sin. ½ *a* is also larger than the sine of *a.* Thus let *a* be 12° : then the nearest multiple of *a* is 84° or 96°, both of which are as far removed as possible from 90°, and the error is as great as possible, and is nearly 1/160th of the whole.

This approximation gives us a very simple construction. Let CM be the required length of the trumpet, and draw ML perpendicular to the axis in O. It is evident that sin. MCO : rad. = MO : CM, and CM or X = \*\*\*\*-—r— = sin. £ *a*

——; but X =—~~CD~~ , and therefore LM equal to CD. 2sin.∣β 2sm.⅞<ι

If therefore the cone be of such a length, that its dia­meter at the month is equal to the length of the part cut off, every line of sound will have at least as many reflec­tions, save one, as if the cone were infinitely long; and the last reflected line will either be parallel to the opposite side of the cone, or lie nearer the axis than this parallel ; con­sequently such a cone will confine all the reflected sounds within a cone whose angle is 2 *a,* and will augment the sound in the proportion of the spherical base of this cone to a complete hemispherical surface. Describe the circle DKT round C, and making DT an arch of 90°, draw the chord DT. Then, since the circles described with the