to our theory of the uniform motion of running waters. But to return to our subject.

Should it be required to bring off at once from the basin a mill-course having a determined velocity for driving an undershot wheel, the problem becomes easier, because the velocity and slope combined determine the hydraulic mean depth at once ; and the depth of the stream will be had by means of the height which must be taken for the whole depth at the entry, in order to produce the required velo­city.

In like manner, having given the quantity to be discharg­ed, and the velocity and the depth at the entry, we can find the other dimensions of the channel ; and the mean depth being found, we can determine the slope.

When the slope of a canal is very small, so that the depth of the uniform stream differs but a little from that at the entry, the quantity discharged is but small. But a great velocity, requiring a great fall at the entry, produces a great diminution of depth, and therefore it may not compensate for this diminution, and the quantity discharged may be smaller. Improbable as this may appear, it is not demon­strably false ; and hence we see the propriety of the follow­ing question.

*Question* 3. Given the depth H at the entry of a rectan­gular canal, and also its width *w ;* required the slope, depth, and velocity which will produce the greatest possible dis­charge.

Let *x* be the unknown depth of the stream. H — *x* is the productive fall, and the velocity is √2G√H*—x.* This multiplied by *w x* will give the quantity discharged. Therefore *wx*√2G√H*—x* must be made a maximum. The common process for this will give the equation 2 H =3 *x,* or *x* = 2/3 H. The mean velocity will be √2G√1/3H ; the section will be 2/3*w* H, and the discharge = 2/3√2G *w*H√½ H, and *d = \*\*\*-3 w,⅞*-τ. With these data the slope is easily had by the formula for uniform motion.

If the canal is of the trapezoidal form, the investigation is more troublesome, and requires the resolution of a cubic equation.

It may appear strange that increasing the slope of a canal beyond the quantity determined by this problem can di­minish the quantity of water conveyed. But one of these two things must happen ; either the motion will not acquire uniformity in such a canal for want of length, or the dis­charge must diminish. Supposing, however, that it could augment, we can judge how far this can go. Let us take the extreme case by making the canal vertical. In this case it becomes a simple weir or waste-board. Now the discharge of a waste-board is \*\*\*\*§ √2 Gw (∕i"—(A Λ)^)∙ The maximum determined by the preceding problem is to that of the waste-board of the same dimensions as \*\*\*H√1/3H : Hi-(⅛H)i or asH√⅜HtH√H-⅜H√⅜H = 5773: 6465, nearly = 9:10.

Having given the dimensions and slope of a canal, we can discover the relation between its expenditure and the time : or we can tell how much it will sink the surface of a pond in twenty-four hours, and the gradual progress of this effect ; and this might be made the subject of a particular problem, but it is complicated and difficult. In cases where this is an interesting object, we may solve the ques­tion with sufficient accuracy, by calculating the expenditure at the beginning, supposing the basin kept full. Then, from the known area of the pond, we can tell in what time this expenditure will sink an inch ; do the same on the supposi­tion that the water is one third lower, and that it is two thirds lower (noticing the contraction of the surface of the

pond occasioned by this abstraction of its waters). Thus we shall obtain three rates of diminution, from which we can easily deduce the desired relation between the expen­diture and the time.

Aqueducts derived from a basin or river are commonly furnished with a sluice at the entry. This changes exceed­ingly the state of things. The slope of the canal may be precisely such as will maintain the mean velocity of the water which passes under the sluice; in which case the depth of the stream is equal to that of the sluice, and the velocity is produced at once by the head of water above it. But if the slope is less than this, the velocity of the issuing water is diminished, and the water must rise in the canal. This must check the efflux at the sluice, and the water will be as it were stagnant above what comes through below it It is extremely difficult to determine at what precise slope the water will begin to check the efflux. The contraction at the lower edge of the board hinders the water from at­taining at once the whole depth which it acquires afterwards, when its velocity diminishes by the obstructions. While the regorging which these obstructions occasion does not reach back to the sluice, the efflux is not affected by it. Even when it does reach to the sluice, there will be a less depth immediately behind it than farther down the canal, where it is in train ; because the swift moving water which is next the bottom drags with it the regorged water which lies on it: but the canal must be rapid to make this differ­ence of depth sensible. In ordinary canals, with moderate slopes and velocities, the velocity at the sluice may be safely taken as if it were that which corresponds to the difference of depths above and below the sluice, where both were in train.

Let therefore H be the depth above the sluice, and *h* the depth in the canal. Let *e* be the elevation of the sluice above the sole, and let *b* be its breadth. The discharge will be *eb* √H*—h* √2G for the sluice, and *wh*√N*g*/√S \*\*\*

√≈ *J* ~~U~~ ~~xt~~ for the canal. These must be the same. This w ∞-∣-2Λ

gives the equation \*\*\**e b./H — hV2* G = *w*

containing the solution of all the questions which can be proposed. The only uncertainty is in the quantity G, which expresses the velocity competent to the passage of the water through the orifice, circumstanced as it is, namely, subject­ed to contraction. This may be regulated by a proper form given to the entry into this orifice. The contraction may be almost annihilated by making the masonry of a cycloidal form on both sides, and also at the lower edge of the sluice-board, so as to give the orifice a form resembling fig. 5, D, in the article River. If the sluice is thin in the face of a basin, the contraction will reduce 2 G to 296. If the sluice be as wide as the canal, 2 G will be nearly 500.

*Question* 4. Given the head of water in the basin H, the breadth *b,* and the elevation *e* of the sluice, and the breadth *w* and slope *s* of the canal, to find the depth *h* of the stream, the velocity, and the discharge.

We must (as in *Question* 2) make a first supposition for *h,* in order to find the proper value of *d.* Then the equa­tion \*\*\*\*\*\*\**ebVH — h* √2 G = *w h* gives λ = *~~θ,~~~~e~~~~^~~~~r~~ ~~$~~* 4-

√s *w2Ngd ' ∕Ge2btSH 1 ∕GeibtS∖t r. ,. , v 11 j.λ,*

*~~< w~tig<l~~* + ΙτνΤλ' ∙ If this value should differ considerably from the one which we assumed in order to begin the computation, make use of it for obtaining a new value of *d,* and repeat the operation. We shall rarely be obliged to perform a third operation.

The following is of frequent use.