best possible state when *a × v* × PS is a maximum. But it is plain that *a × υ* is an invariable quantity, for it is the cubic inches of water which the spout supplies in a second. If the wheel moves fast, little water lies in each bucket, and *a* is small. When *ν* is small, *a* is great, for the opposite reason ; but *a × v* remains the same. Therefore we must make PS a maximum, that is, we must deliver the water as high up as possible. But this diminishes AP, and this di­minishes the velocity of the wheel ; and as this has no limit, the proposition is demonstrated, and an overshot wheel does the more work as it moves slowest.

Convincing as this discussion must be to any mechani­cian, we are anxious to impress the same maxim on the minds of practical men, unaccustomed to mathematical reasoning of any kind. We therefore beg indulgence for adding a popular view of the question, which requires no such investigation.

We may reason in this way. Suppose a wheel having thirty buckets, and that six cubic feet of water are deliver­ed in a second on the top of a wheel, and discharged with­out any loss by the way at a certain height from the bot­tom of the wheel. Let this be the case whatever is the rate of the wheel’s motion, the buckets being of a sufficient capacity to hold all the water which falls into them. Let this wheel be employed to raise a weight of any kind, sup­pose water in a chain of thirty buckets, to the same height and with the same velocity. Suppose further, that when the load on the rising side of the machine is one half of that on the wheel, the wheel makes four turns in a minute, or one turn in fifteen seconds. During this time ninety cubic feet of water have flowed into the thirty buckets, and each has received three cubic feet. Then each of the rising buckets contains 1½ feet, and forty-five cubic feet are de­livered into the upper cistern during one turn of the wheel, and 180 cubic feet in one minute.

Now, suppose the machine so loaded, by making the rising buckets more capacious, that it makes only two turns in a minute, or one turn in thirty seconds. Then each de­scending bucket must contain six cubic feet of water. If each bucket of the rising side contained three cubic feet, the motion of the machine would be the same as before. This is a point which no mechanician will controvert. When two pounds are suspended to one end of a string which passes over the pulley, and one pound to the other end, the descent of the two pounds will he the same with that of a four pound weight which is employed in the same manner to draw up two pounds. Our machine would therefore con­tinue to make four turns in the minute, and would deliver ninety cubic feet during each turn, and 360 in a minute. But, by supposition, it is making but two turns in a minute: this must proceed from a greater load than three cubic feet of water in each rising bucket. The machine must there­fore be raising *more* than ninety feet of water during one turn of the wheel, and *more* than 180 in the minute.

Thus it appears, that if the machine be turning twice as slow as before, there is *more than twice the former quantity* in the rising buckets, and more will be raised in a minute by the same expenditure of power. In like manner, if the machine go three times as slow, there must be *more than three times* the former quantity of water in the rising buckets, and more work will be done.

But we may go further, and assert, that the *more* we re­tard the machine, by loading it with more work of a simi­lar kind, the greater will be its performance. This does not immediately appear from the present discussion : but let us call the first quantity of water in the rising bucket A ; the water raised by four turns in a minute will be 4 × 30 × A = 120 A. The quantity in this bucket, when the machine goes twice as slow, has been shown to be greater than 2 A (call it 2 A + *x*) ; the water raised by two turns in a minute will be 2 × 30 × (2 A + *x*) *=* 120 A + 60 *x.* Now, let the machine go four times as slow, making but one turn in a minute, the rising bucket must now contain more than twice 2 A -∣- *x,* or more than 4 A + 2 *x ;* call it 4 A + 2 *x + y.* The work done by one turn in a minute will now be 30 × (4 *A* + 2 *x* + *y) =* 120 A + 60*x* + 30 *y.*

By such an induction of the work, done with any rates of motion we choose, it is evident that the performance of the machine increases with every diminution of its velocity that is produced by the mere addition of a similar load of work, or that it does the more work the slower it goes.

We have supposed the machine to be in its state of perma­nent uniform motion. If we consider it only in the begin­ning of its motion, the result is still more in favour of slow motion : for, at the first action of the moving power, the inertia of the machine itself consumes part of it, and it ac­quires its permanent speed by degrees, during which the resistances arising from the work, friction, &c. increase, till they exactly balance the pressure of the water ; and after this the machine accelerates no more. Now the greater the power and the resistance arising from the work are in proportion to the inertia of the machine, the sooner will all arrive at its state of permanent velocity.

There is another circumstance which impairs the performance of an overshot wheel moving with a great velocity, viz. the effects of the centrifugal force on the water in the buckets. Our mill­wrights know well enough, that too great velocity will throw the water out of the buckets ; but few, if any, know exactly the diminution of power produced by this cause. The following very simple construction will determine this. Let AOB (fig. 10) be an overshot wheel, of which AB is the upright diameter, and C is the centre. Make CF the length of a pendulum which will make two vibrations during one turn of the wheel. Draw FE to the elbow of any of the buckets. The water in this bucket, instead of having its surface ho­rizontal, as NO, will have it in the di­rection *n*O perpendicular to FE very nearly.

For the time of falling along half of FC is to that of two vibrations of this pendulum, or to the time of a revolution of the wheel, as the radius of a circle is to its circumfe­rence ; and it is well known that the time of moving along half of AC, by the uniform action of the centrifugal force, is to that of a revolution as the radius of a circle to its cir­cumference. Therefore the time of describing one half of AC by the centrifugal force, is equal to the time of describ­ing one half of FC by gravity. These spaces, being simi­larly described in equal times, are proportional to the acce­lerating forces. Therefore ½ FC : ½ AC, or FC : AC = gra­vity : centrifugal force. Complete the parallelogram FCEK. A particle at E is urged by its weight in the direction KE with a force which may be expressed by FC or KE ; and it is urged by the centrifugal force in the direction CE, with a force = AC or CE. By their combined action it is urged in the direction FE. Therefore, as the surface of standing water is always at right angles to the action of gravity, that is, to the plumb-line, so the surface of the water in the revolving bucket is perpendicular to the action of the combined force FE.

Let NEO be the position of the bucket, which just holds all the water which it received as it passed the spout when not affected by the centrifugal force; and let NDO be its position when it. would be empty. Let the vertical lines through D and E cut the circle described round C with the