radius CF in the points II and I. Draw HC, IC, cutting the circle AOB in L and Μ. Make the arch *d'δ* equal to AL, and the arch *e'ε* equal to AM : then C*δ* and C*ε* will be the positions of the bucket on the revolving wheel, cor­responding to CD*o* and CEO on the wheel at rest. Wa­ter will begin to run out at ε, and it will be all gone at *δ*. The demonstration is evident.

The force which now urges the wheel is still the weight *really* in the buckets ; for though the water be urged in the direction with the force FE, one of its constituents, CE, has no tendency to impel the wheel ; and KE is the only impelling force.

It is but of late years that mills have been constructed or attended to with that accuracy and scientific skill which are necessary for deducing confident conclusions from any experiments that can be made with them ; and it is therefore no matter of wonder that the opinions of mill­wrights have been so different on this subject. There is a natural wish to see a machine moving briskly ; it has the appearance of activity ; but a very slow motion always looks as if the machine were overloaded. For this reason mill-wrights have always yielded slowly, and with some re­luctance, to the repeated advices of the mathematicians : but they have yielded, and we see them adopting maxims of construction more agreeable to sound theory, making their wheels of great breadth, and loading them with a great deal of work. Euler says that the performance of the best mill cannot exceed that of the worst above one fifth ; but we have seen a stream of water completely ex­pended in driving a small flax-mill, which now drives a cot­ton-mill of 4000 spindles, with all its carding, roving, and drawing machinery, besides the lathes and other engines of the smith and carpenter’s workshops, exerting a force not less than ten times what sufficed for the flax-mill.

The above discussion only demonstrates in general the advantage of slow motion, but does not point out in any degree the relation between the rate of motion and the work performed, nor even the principles on which it de­pends. Yet this is a subject fit for a mathematical inves­tigation, and we would prosecute it in this place if it were necessary for the improvement of practical mechanics. But we have seen that there is not, in the nature of things, a maximum of performance attached to any particular rate of motion which should therefore be preferred. For this reason we omit this discussion of mere speculative curio­sity. It is very intricate; for we must now express the pressure on the wheel by a *constant* pillar of water incum­bent on the extremity of the horizontal arm, as we did be­fore when we supposed the buckets completely filled ; nor by a smaller *constant* pillar, corresponding to a smaller but equal quantity lying in every bucket. Each different ve­locity puts a different quantity of water into the bucket as it passes the spout, and this occasions a difference in the place where the discharge is begun and completed. This circumstance is some obstacle to the advantages of very slow motions, because it brings on the discharge sooner. All this may indeed be expressed by a simple equation of easy management ; but the whole process of the mecha­nical discussion is both intricate and tedious, and the re­sults are so much diversified by the forms of the buckets, that they do not afford any rule of sufficient generality to reward our trouble. The curious reader may see a very full investigation of this subject in two dissertations by Elvius in the Swedish Transactions, and in the *Hydrody­namique* of Professor Kästner of Göttingen, who has abridged these dissertations of Elvius, and considerably improved the whole investigation, and has added some com­parisons of his deductions with the actual performance of some great works. These comparisons however arc not very satisfactory. There is also a very valuable paper on this subject by Lambert, in the Memoirs of the Academy of

Berlin for the year 1775. From these dissertations, and from the *Hydrodynamique* of the Abbé Bossut, the reader will acquire all that theory can teach of the relation be­tween the pressures of the power and work on the machine, and the rates of its motion. The practical reader may rest with confidence on the simple demonstration which we have given, that the performance is improved by diminishing the velocity.

All we have to do, therefore, is to load the machine, and thus to diminish its speed, unless other physical circum­stances throw obstacles in the way ; but there are such ob­stacles. In all machines there are little inequalities of ac­tion that are unavoidable. In the action of a wheel and pinion, though made with the utmost judgment and care, there are such inequalities. These increase by the changes of form occasioned by the wearing of the machine : much greater irregularities arise from the subsultory motions of cranks, stampers, and other parts which move unequally or reciprocally. A machine may be so loaded as just to be in equilibrio with its work, in the favourable position of its parts. When this changes into one less favourable, the machine may stop ; if not, it at least staggers, hobbles, or works unequally. The rubbing parts bear long on each other, with enormous pressures, and cut deep, and increase friction. Such slow motions must therefore be avoided. A little more velocity enables the machine to get over those increased resistances by its inertia, or the great quantity of motion inherent in it. Great machines possess this advan­tage in a superior degree, and will therefore work steadily with a smaller velocity. These circumstances are hardly susceptible of mathematical discussion, and our best reli­ance is on well-directed experience.

For this purpose, the reader will do well to peruse with care the excellent paper by Mr Smeaton in the Philoso­phical Transactions for 1759. This dissertation contains a numerous list of experiments, most judiciously contrived by him, and executed with the accuracy and attention to the most important circumstances which is to be observed in all that gentleman’s performances.

It is true, these experiments were made with small mo­dels ; and we must not, without great caution, transfer the results of such experiments to large works. But we may safely transfer the laws of variation which result from a variation of circumstances, although we must not adopt the absolute quantities of the variations themselves. Mr Smeaton was fully aware of the limitations to which con­clusions drawn from experiments on models are subject, and has made the applications with his usual sagacity.

His general inference is, that, in smaller works, thc rim of the overshot wheel should not have a greater velocity than three feet in a second ; but that larger mills may be allowed a greater velocity than this. When every thing is executed in the best manner, he says that the work per­formed will amount to fully two thirds of the power ex­pended ; that is, that three cubic feet of water descending from any height will raise two to the same height.

It is not very easy to compare these deductions with ob­servations on large works ; because there are few cases where we have good measures of the resistances opposed by the work performed by the machine. Mills employed for pumping water afford the best opportunities. But the inertia of their working gear diminishes their useful per­formance very sensibly, because their great beams, pump- rods, &c. have a reciprocating motion, which must be de­stroyed and produced anew in every stroke. We have examined some machines of this kind which are esteemed good ones, and we find few of them whose performance exceeds one half of the power expended.

By comparing other mills with these, we obtain the best information of their resistances. The comparison with mills worked by Watt and Boulton’s steam-engines, is per-