of book X. on incommensurables and props. 2 and 16 of book xii., viz., that “circles are to one another as the squares on their diameters” and that “in the greater of two concentric circles a regular 2*n*-gon can be inscribed which shall not meet the circumference of the less,” how­ever nearly equal the circles may be. With Archimedes (287-212 b.c.) a notable advance was made. Taking the circumference as intermediate between the perimeters of the inscribed and the circumscribed regular n-gons, he showed that, the radius of the circle being given and the perimeter of some particular circumscribed regular polygon obtainable, the perimeter of the circumscribed regular polygon of double the number of sides could be calculated ; that the like was true of the inscribed polygons; and that consequently a means was thus afforded of approximating to the circumference of the circle. As a matter of fact, he started with a semi-side AB of a circumscribed regular hexagon meeting the circle in B (see fig. 1), joined A and B with O the centre, bisected the AOB by OD, so that BD became the semi-side of a circumscribed regular 12-gon ; then as AB : BO : OA : : 1 : √3 : 2 he sought an approximation to √3 and found that AB : BO >153 : 265. Next he applied his theorem@@1 BO + OA : AB : : OB : BD to calculate BD ; from this in turn he cal­

culated the semi-sides of the circumscribed

regular 24-gon, 48-gon, and 96-gon, and

so finally established for the circumscribed

regular 96-gon that perimeter : diameter

< 31/7 : 1. In a quite analogous manner he proved for the inscribed regular 96-gon that perimeter : diameter > 310/71 : 1. The conclusion from these therefore was that the ratio of cir­cumference to diameter is <31 and >3110/71. This is a most notable piece of work ; the immature condition of arith­metic at the time was the only real obstacle preventing the evaluation of the ratio to any degree of accuracy whatever.@@2

No advance of any importance was made upon the achievement of Archimedes until after the revival of learn­ing. His immediate successors may have used his method to attain a greater degree of accuracy, but there is very little evidence pointing in this direction. Ptolemy (fl. 127- 151), in the *Great Syntaxis,* gives 3·141552 as the ratio@@3; and the Hindus (c. 500 a.d.), who were very probably indebted to the Greeks, used 62832/20000, that is, the now familiar 3·1416.@@4 It was not until the 15th century that attention in Europe began to be once more directed to the subject, and after the resuscitation a considerable length of time elapsed before any progress was made. The first advance in accuracy was due to a certain Adrian, son of Anthony, a native of Metz (1527), and father of the better- known Adrian Metius of Alkmaar. In refutation of Duchesne (Van der Eycke) he showed that the ratio was

< 317/120 and > 315/106, and thence made the exceedingly lucky step of taking a mean between the two by the quite unjusti­fiable process of halving the sum of the two numerators for a new numerator and halving the sum of the two denomi­nators for a new denominator, thus arriving at the now well-known approximation 316/113 or 355/113 which, being equal to 3·1415929..., is correct to the sixth fractional place.@@5 The next to advance the calculation was Viète (De Viette, Vieta), the greatest mathematician of his age. By finding the perimeter of the inscribed and that of the circumscribed regular polygon of 393216 *(i.e.,* 6 × 216) sides, he proved

that the ratio was >3∙1415926535 and <3T415926537, so that its value became known (in 1579) correctly to 10 fractional places. The theorem for angle-bisection which Viète used was not that of Archimedes, but that which would now appear in the form 1 - cos*θ* = 2sin2 ½*θ*

. With Viète, by reason of the advance in arithmetic, the style of treatment becomes more strictly trigonometrical ; in­deed, the *Universales Inspectiones,* in which the calculation occurs, would now be called plane and spherical trigono­metry, and the accompanying *Canon Mathematicus,* a table of sines, tangents, and secants.@@6 Further, in comparing the labours of Archimedes and Viète, the effect of increased power of symbolical expression is very noticeable. Archi­medes’s process of unending cycles of arithmetical opera­tions could at best have been expressed in his time by a “rule” in words; in the 16th century it could be condensed into a “ formula.” Accordingly, we find in Viète a formula for the ratio of diameter to circumference, viz., the intermi­nate product@@7—

⅜V⅛∙λ∕ ⅛ + ⅛√⅛∙ n∕4 + ⅛√⅛ + ⅛V⅛∙ ■ ·

From this point onwards, therefore, no knowledge what­ever of geometry was necessary in any one who aspired to determine the ratio to any required degree of accuracy : the mere arithmetician’s art and length of days were the only requisites. Thus in connexion with the subject a genus of workers became possible who may be styled “π-computers,”—a name which, if it connotes anything uncomplimentary, does so because of the almost entirely fruitless character of their labours. Passing over Adriaan van Roomen (Adrianus Romanus) of Louvain, who pub­lished the value of the ratio correct to 15 places in his *Idea Mathematica* (1593),@@8 we come to the notable com­puter Ludolph van Ceulen (d. 1610), a native of Germany, long resident in Holland. His book, *Van den Circkel* (Delf, 1596), gave the ratio correct to 20 places, but he continued his calculations as long as he lived, and his best result was published on his tombstone in St Peter’s church, Leyden. The inscription, which is not known to be now in existence,@@9 is in part as follows :—

. . . Qui in vita sua multo labore circumferentiae circuli proxi­mam rationem ad diametrum invenit sequentem—

quando diameter est 1 tum circuli circumferentia plus est

314159265358979323846264338327950288 quam 100000000000000000000000000000000000

et minus

314159265358979323846264338327950289 quam 100000000000000000000000000000000000...

This gives the ratio correct to 35 places. Van Ceulen’s process was essentially identical with that of Viète. Its numerous root extractions amply justify a stronger expres­sion than “ multo labore,” especially in an epitaph. In Germany the “Ludolphische Zahl” is still a common name for the ratio.@@10

Up to this point the credit of most that had been done may be set down to Archi­medes. A new departure, however, was made by Willebrord Snell of Ley­den in his *Cyclometria,* published in 1621. His achievement was a closely approximate geometrical solu­

@@@1 In modem trigonometrical notation, 1 + sec θ : tan ½

: : 1 : tan ½θ.

@@@2 Tannery, “ Sur la mesure du cercle d’Archimède, ” in Mém. . . . Bordeaux, [2], iv. pp. 313-339 ; Menge, Des Archimedes Kreismessung, Coblentz, 1874.

@@@3 De Morgan, in Penny Cyclop., xix. p. 186.

@@@4 Kern, Aryahhattîyam, Leyden, 1874, trans, by Rodet, Paris, 1879.

@@@5 De Morgan, art. “Quadrature of the Circle,” in English Cyclop. ;

Glaisher, Mess. of Math., ii. pp. 119-128, iii. pp. 27-46 ; De Haan, Nieuw Archief V. Wisk., i. pp. 70-86, 206-211.

@@@6 Vieta, *Opera Math.,* Leyden, 1646; Marie, *Hist. des Sciences Math.,* iii. p. 27 *sq.,* Paris, 1884.

@@@7 Klügel, *Math. Wörterb.,* ii. pp. 606, 607.

@@@8 Kästner, *Gesch, d. Math.,* i., Göttingen, 1796-1800.

@@@9 But see *Les Délices de Leide,* Leyden, 1712 ; or De Haan, *Mess. of Math.,* iii. pp. 24-26.

@@@10 For minute and lengthy details regarding the quadrature of the circle in the Low Countries, see De Haan, “ Bouwstoffen voor de geschie­denis, &c.,” in *Versl. en Mededeel. der K. Akad. van Wetensch.,* ix., x., xi., xii., Amsterdam ; also his “ Notice sur quelques quadrateurs, &c.,” in *Bull di Bibliogr. e di Storia delle Sci. Mat. e Fis.,* vii. pp. 99-144.